RAM Algorithms 2

AVA Fall 2005, week 9

Content: Predecessor problem, van Emde Boas priority queues, y-fast tries.

Exercises: Discuss some solutions to the following problems with their space and time complexity for each operation. Also, identify the (information-theoretic) space lower bound for each problem.

- **Partial sums:** Given a sequence $x_1, \ldots, x_n$ of n elements, support the following operations, under the restriction that the sum of all element at any instance is at most m:
  - $sum(i)$: return $\sum_{j=1}^{i} x_j$
  - $update(i, k)$: $x_i \leftarrow x_i + k$
- **Bit vector with rank and select:** Given a bit vector B of length m, support the following operations:
  - $rank(i)$: return the number of 1's upto position i in B
  - $select(j)$: return the position of the j-th 1 in B
- **Subset with rank and select:** Given a subset $S \subseteq \{0, 1, \ldots, m-1\}$, with $|S| = n$, support the following operations:
  - $rank(x)$: return $|\{y \in S | y \leq x\}|$
  - $select(i)$: return the i-th element in the sorted order of S
- **Dynamic range reporting in one dimension:** Given a set of n integers from the universe $\{0, \ldots, u-1\}$, maintain it under the following oprations:
  - $report(i, j)$: report an element from the set in the interval $[i, j]$
  - $insert/delete(x)$: insert/delete the element x to/from the set

Mandatory assignment:

1. Describe the procedures to support the $Successor$, $Insert$, $Delete$ and $ExtractMin$ operations using the van Emde Boas structure.
2. The van Emde Boas structure mentioned in the class requires $O(m \lg m)$ bits of space. Describe how the space can be improved to $O(m)$ bits, without changing the query times.