Introduction to Algorithms and Data Structures E2004 : More Homework Exercises, Episode 6, October 4, 2004

1. Recall the Fibonacci numbers: $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_n + F_{n+1}$. Prove by induction on n that $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$.

2 **(M)**. (a) Show the AVL trees resulting from successively inserting the keys 41, 38, 31, 12, 19, 8, 27, 49 (in this order) into an initially empty AVL tree, rebalancing after each insertion.
(b) Indicate the balances of the nodes in the resulting AVL tree.
(c) Now delete the keys 12, 49, 31, 8 (in this order), showing the AVL tree after each deletion and rebalancing.

3 **(H)**. Prove that after deleting an item from an AVL tree T and rebalancing, the height of the resulting tree T' does not exceed that of T.

4 **(M)**. Give an illustrated example of an AVL tree T and a new key x such that the successive operations of inserting x into T, rebalancing, removing the key x from the tree, and rebalancing again lead to an AVL tree T' different from T.

5. Describe an algorithm (in pseudocode or plain English) for the deletion of a given node from an AVL tree and successive rebalancing.

6. Show that any n-node binary tree can be transformed into any other n-node binary tree by using $O(n)$ (single) rotations.

   [Hint: First show that at most $n - 1$ single clockwise rotations suffice to transform the tree into a right-going chain.]

7 **(D)**. For any node x in a binary tree, let l_x be 1 + the length of the longest path from x to some leaf below x, and let s_x be 1 + the length of the shortest path from x to some leaf below x. Prove the following property of AVL trees: $l_x \leq 2 \cdot s_x$ for any node x.