

Important sums to know in IADS

1. Sum

$$\sum_{i=0}^n 2^i = 1 + 2 + 4 + 8 + \dots + 2^n = 2^{n+1} - 1 = O(2^n)$$

an important instance

$$\sum_{i=0}^{\lceil \lg n \rceil} 2^i = 1 + 2 + 4 + 8 + 16 + \dots + 2^{\lceil \lg n \rceil} = 2^{\lceil \lg n \rceil + 1} - 1 = O(n)$$

this is also the case if you replace $\lceil \lg n \rceil$ by $\lfloor \lg n \rfloor$

2. Sum

$$\sum_{i=0}^n \left(\frac{1}{2}\right)^i = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 2 - \frac{1}{2^n} = O(1)$$

3. Sum

$$\sum_{i=1}^n i = 1 + 2 + 3 + 4 + \dots + n = \frac{(n+1)n}{2} = O(n^2)$$

These sums can often appear in the opposite direction:

Example: $2^n + 2^{n-1} + \dots + 1 = 2^{n+1} - 1 = O(2^n)$

They can also appear multiplied or divided by a constant:

For instance is the 2. sum just the first rule divided by 2^n and written in the opposite direction.