

SASP/AMP F2011 - Assignment 10:

Data-Flow Analysis

Deadline: Wednesday April 13 at 09:00 (hand it in at the lecture or by email before).

(I recommend completing it before Monday's lecture, as we will build on much of this knowledge.)

1) Undecidability:

Prove that the following problem is *undecidable* (using the “reduction principle”):

- *what are the possible outputs of a program 'P'?*

Let's assume output is done via a special statement (the syntax of which is):

```
STM ::= output EXP1 ";"
```

In addition to carrying out the reduction, you need to explain your reasoning. (Hint: it's quite similar to the examples you saw on slides #18+#20 at the lecture.) :-)

2) Control-Flow Diagrams:

Give a *control-flow template* (as the ones on slides #35+#36) for the “&&”-construction (aka., “lazy conjunction”):

```
EXP ::= EXP1 "&&" EXP2
```

You need to *strictly* adhere to the conventions (of drawing)....:

- **statements** as rectangles (with flow *in* and *out*);
- **expressions** (of type non-boolean) as rectangles (with flow *in* and *out*);
- **expressions** (of type boolean) as diamonds (with single flow in and with boolean flow out as two distinct paths, one for “true” and one for “false”); *and*
- **confluence** drawn explicitly as circles (collecting multiple flows of control).

3) Control-Flow Graphs:

Draw a *control-flow graph* for the following (silly) program fragment:

```
int N = 5;
int x=input();
int y=input();
for (int i=1; i<N; i++) {
  if (y!=0 && x/y>2) x = x+1;
  else {
    y = y-1;
    while (x>10) x = x/2;
  }
}
output x;
```

(Note: the program isn't supposed to do anything remotely interesting.)

4) Relations and Partial-Orders:

Consider the *subset-of* relation over the set $S = \mathcal{P}(\{x+1, 2*y, z/3\})$ of expressions in a program (written " $X \subseteq Y$ " if X is a subset of Y , in short-hand notation). We'd need such a structure in an analysis that tracks "expressions" (e.g., "very busy expressions"-analysis that tracks which expressions have already been computed and haven't changed since).

Give:

- its signature;
- the relation (specify its members);
- an example of a member of the relation (both w/ and w/o using short-hand); *and*
- an example of a non-member of the relation (w/ and w/o using short-hand).

Does the set S and relation form a partial-order? (why or why not?)

Draw a Hasse diagram.

5) Greatest-Lower-Bound:

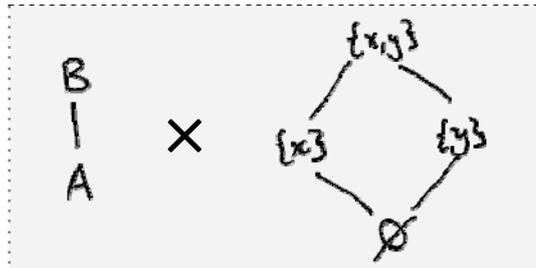
Define the *greatest-lower-bound* (binary operator) on sets ' \sqcup ' which is analogous to the "least-upper-bound" (binary operator): ' \sqcap ' (cf. slide #16 from the 2nd lecture).

Note: it must be: i) *an* lower bound and ii) *the* (i.e., unique) *greatest* lower bound.

Given a lattice $\mathcal{L} = (S, \subseteq)$; what do the elements ' $\sqcup S$ ' and ' $\sqcap S$ ' correspond to?

6) Lattices:

Draw the lattice:



We define the size of a lattice $|\mathcal{L}|$ as how many elements it has.

In general; how many points will a lattice $\mathcal{L}_1 \times \mathcal{L}_2$ have (assuming \mathcal{L}_1 has $|\mathcal{L}_1| = n_1$ elements and \mathcal{L}_2 has $|\mathcal{L}_2| = n_2$ elements)?

7) Monotone Functions and Fixed-Points:

For each of the 3 recursive equations (over the power-lattice: $\mathcal{P}(\{a,b,c\})$):

i)
$$\begin{aligned} X &= \{a,b\} \\ Y &= X \cup Y \end{aligned}$$

ii)
$$\begin{aligned} X &= \{a,b\} \cup Y \\ Y &= X \setminus \{b\} \end{aligned}$$

iii)
$$\begin{aligned} X &= \{a,b\} \cup Z \\ Y &= \{a,c\} \setminus X \\ Z &= X^c \end{aligned}$$

Rewrite the equations to bring them onto form: " $x = f(x,y)$ " and " $y = g(x,y)$ ". Determine whether or not the functions (i.e., ' f ' and ' g ') involved are monotone.

Then, solve the equations that only use monotone functions (i.e., find the [unique] *least fixed point* using the fixed-point theorem).