Exercise 1

Consider the category $\cdot \to \cdot$ consisting of two objects and one non-trivial morphism between them, and consider the functor $\Delta\colon \mathbf{Set} \to \mathbf{Set}^{\cdot \to \cdot}$ mapping X to the constant functor to X. Construct left and right adjoints to Δ and show that these functors are representable.

Exercise 2

Let A be a set. Consider the equational theory T_{print} consisting of operations print_a of arity 1 for each element $a \in A$, and no equations. This is the equational theory for printing. Show that the category of models $\mathrm{Mod}(T_{\mathrm{print}}, \mathbf{Set})$ for the theory is equivalent to the category of left A^* sets defined as follows. Let A^* be the free monoid on A. A left A^* set is a set X with a left A^* action, i.e. a map $\cdot\colon A^* \times X \to X$ such that $a \cdot (a' \cdot x) = (a \cdot a') \cdot x$ and $() \cdot x$ where $a \cdot a'$ is the multiplication of the monoid and $()$ is the unit. A morphism of left A^* sets is a map f of the underlying sets such that

$$\begin{array}{ccc} A^* \times X & \xrightarrow{\mathrm{id} \times f} & A^* \times Y \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & Y \end{array}$$

commutes.

Describe the left adjoint to the forgetful functor $\mathrm{Mod}(T_{\mathrm{print}}, \mathbf{Set}) \to \mathbf{Set}$.