Advanced Database Technology
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INDEXING II

Lecture based on [GUW, 13.3-13.4] and [Pagh03, 3.0-3.2+4]

Slides based on
Notes 04: Indexing
Notes 05: Hashing and more
for Stanford CS 245, fall 2002
by Hector Garcia-Molina

Today

- Recap of indexes
- B-trees
- Analysis of B-trees
- B-tree variants and extensions
- Hash indexes

Why indexing?

- Support more efficiently queries like:
  SELECT * FROM R WHERE a=11
  SELECT * FROM R WHERE 0<= b and b<42
- Indexing an attribute (or set of attributes) can be used to speed up finding tuples with specific values.

Indexes in last lecture

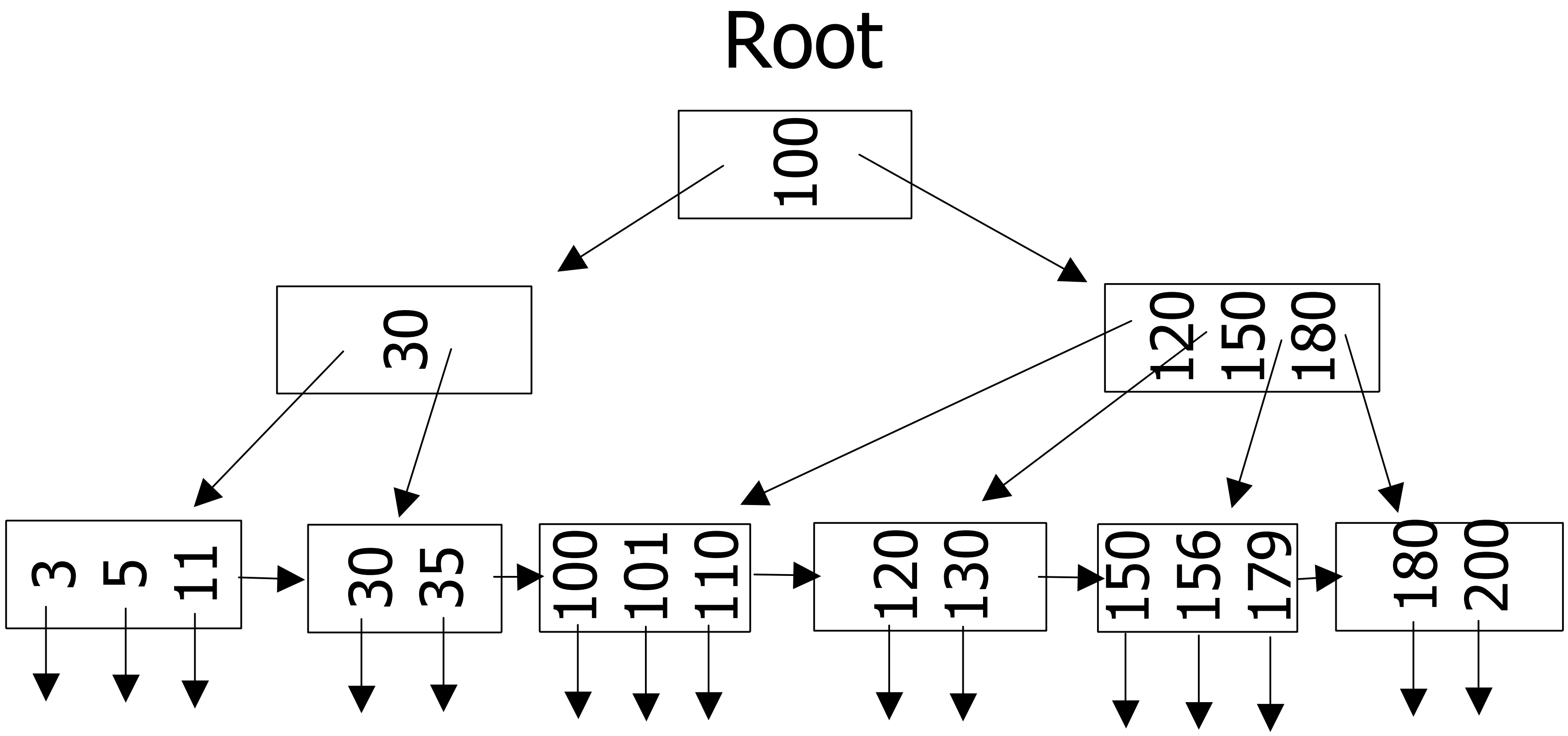
- Dense indexes (primary or secondary)
- Sparse indexes (always primary)
- Clustered indexes (always primary)
- Multi-level indexes

- **Updates** (inserting or deleting a key) caused problems

B-trees

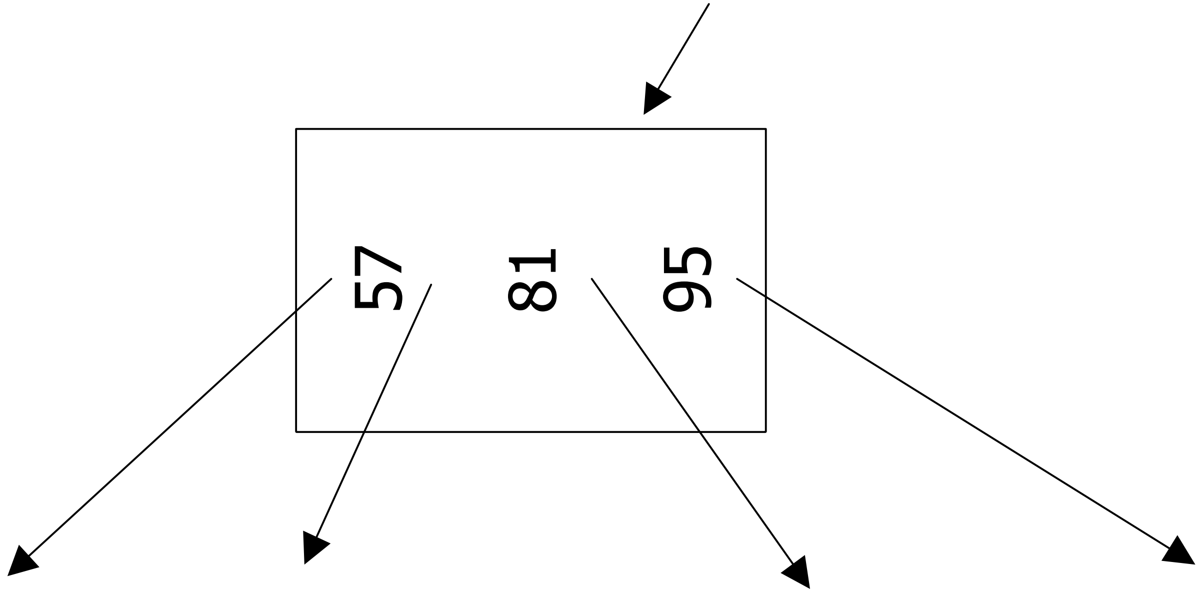
- Can be seen as a general form of multi-level indexes.
- Generalize usual (binary) search trees. *(Do you remember?)*
- Allow efficient insertions and deletions at the expense of using slightly more space.
- Popular variant: B^+-tree

B$^+$-tree Example



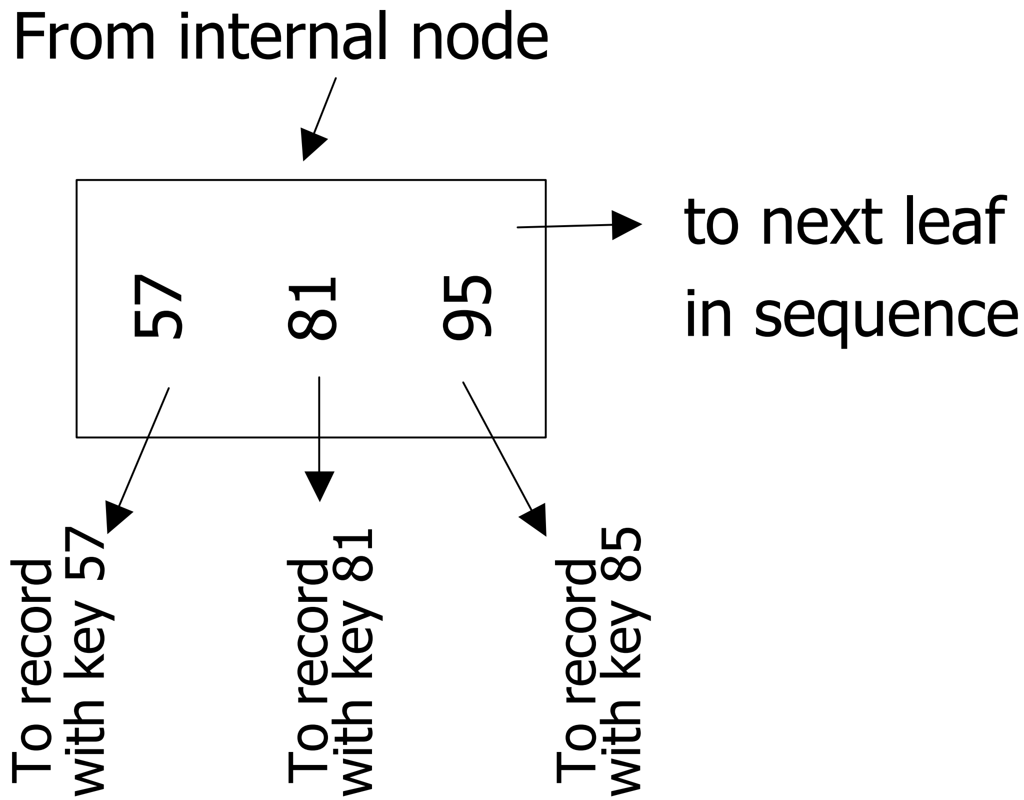
Each node stored in one disk block

Sample internal node



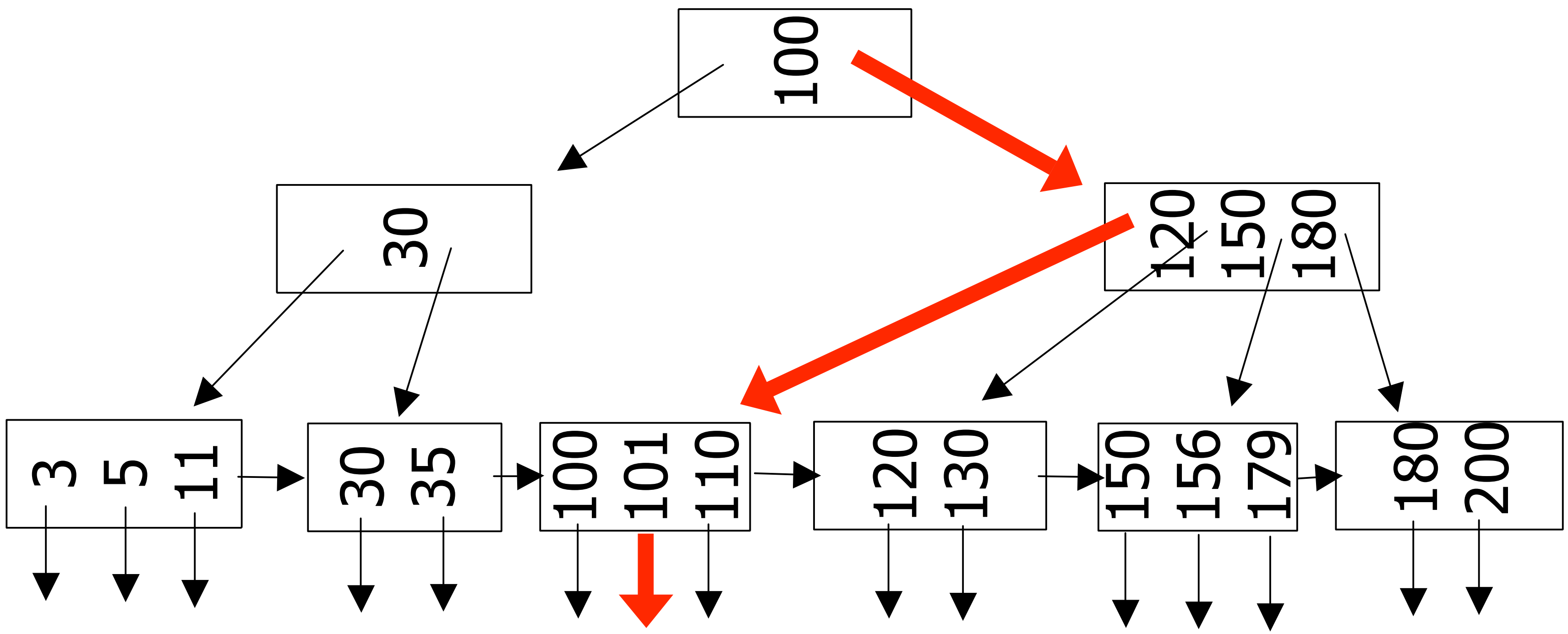
to keys	to keys	to keys	to keys
< 57	57≤ k<81	81≤k<95	≥95

Sample leaf node:



Alternative: Records in leaves

Searching a B$^+$-tree

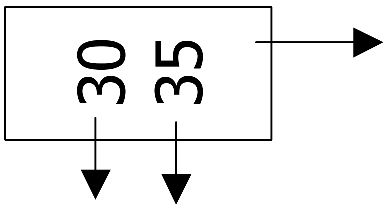


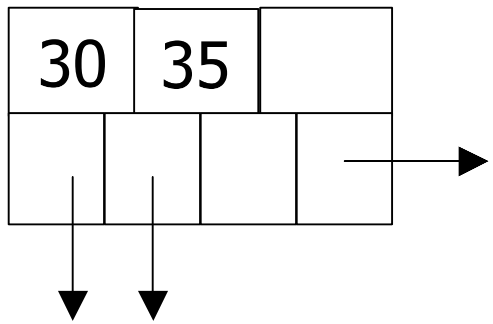
Above: Search path for tuple with key 101.

Question: How does one search for a **range** of keys?

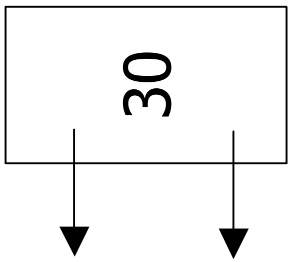
In textbook's notation

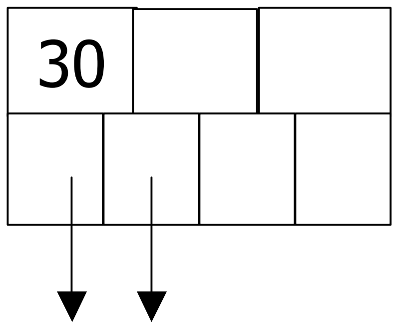
Leaf:





Internal node:





B^+-tree invariants on nodes

- Suppose a node (stored in a block) has space for ***n*** keys and **n+1** pointers.
- Don't want block to be too empty: Should have at least $\lfloor$**(n+1)/2**$\rfloor$ non-null pointers.
- <u>Exception</u>: The root, which may have only 2 non-null pointers.

Other B^+-tree invariants

(1) All leaves at same lowest level (perfectly balanced tree)

(2) Pointers in leaves point to records except for *sequence pointer*

Insertion into B^+-tree

(a) simple case

- space available in leaf

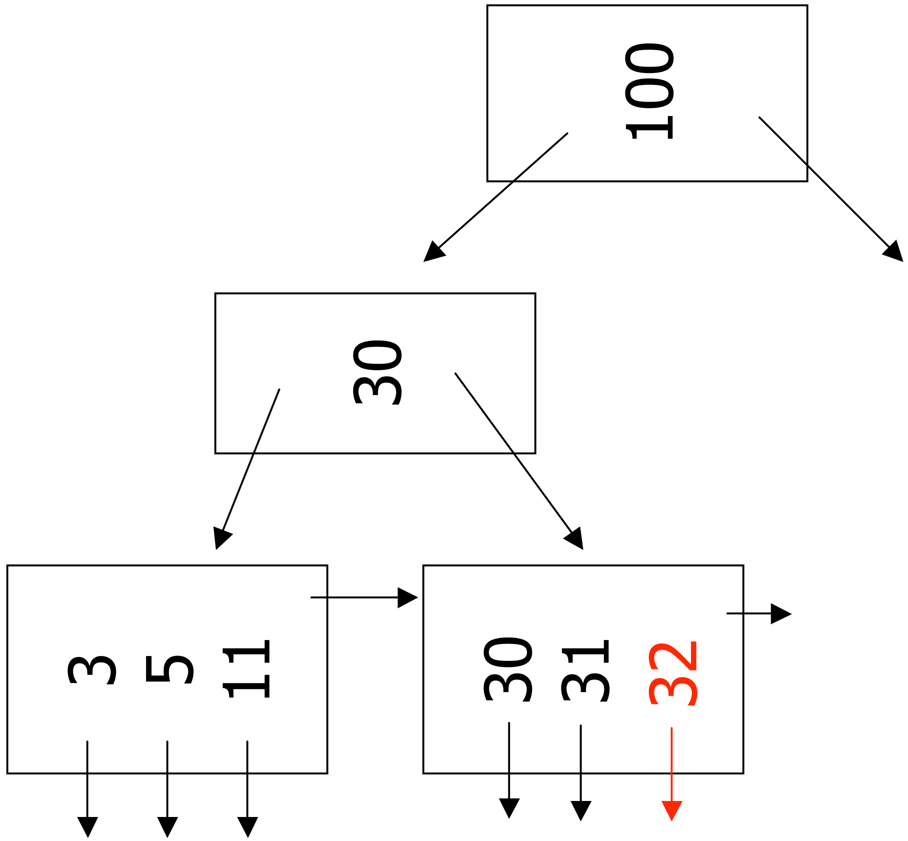
(b) leaf overflow

(c) non-leaf overflow

(d) new root

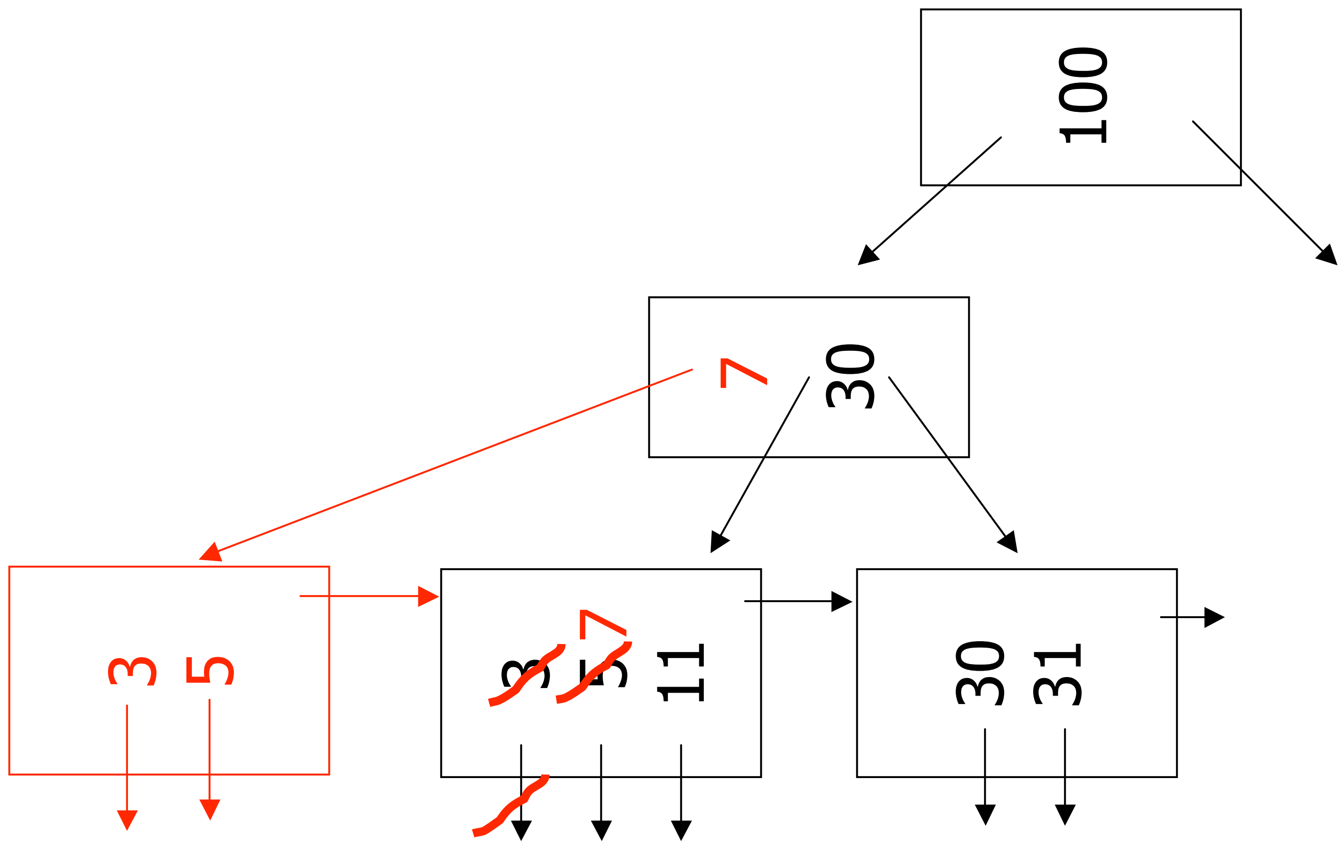
(a) Insert key = 32

n=3



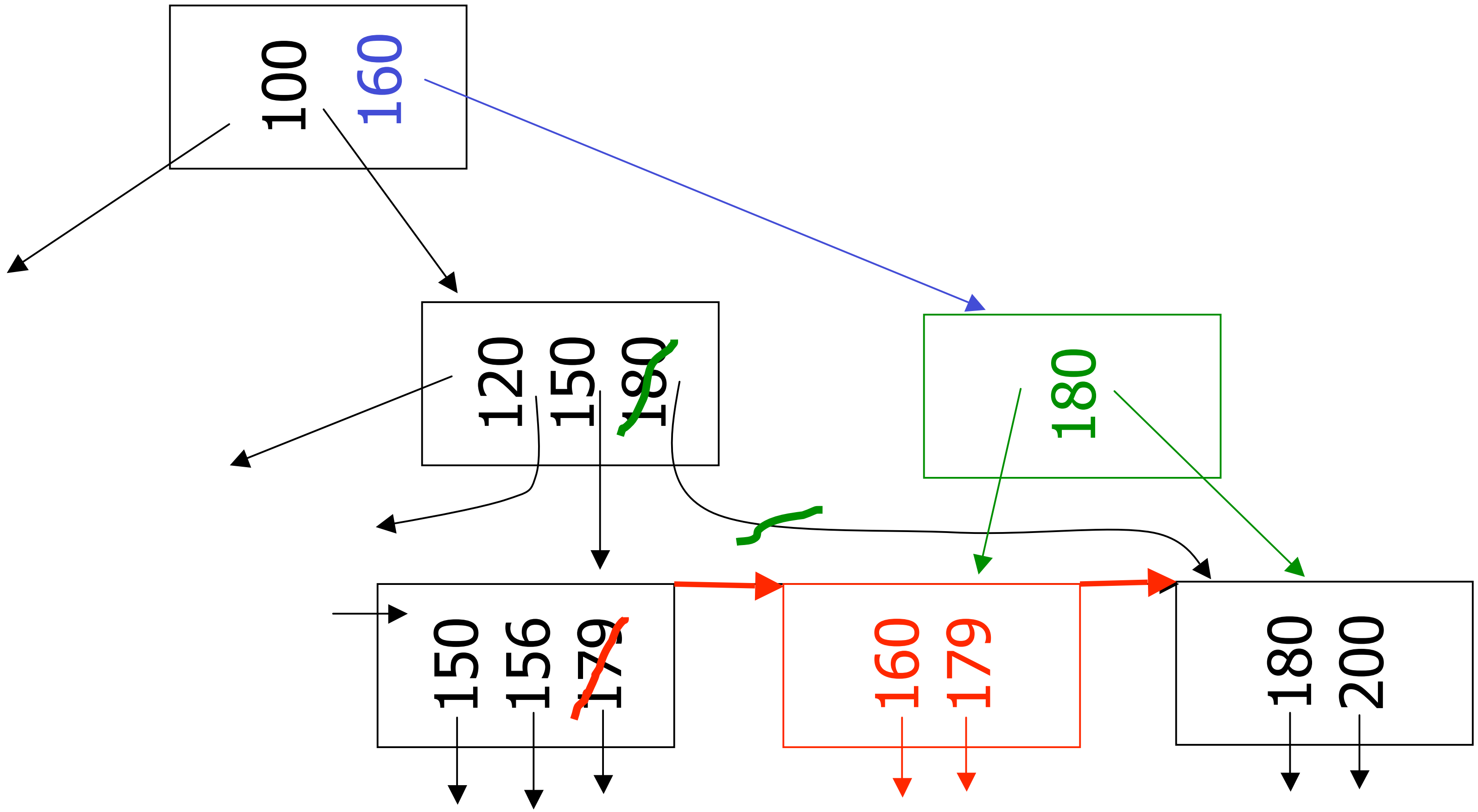
(b) Insert key = 7

n=3



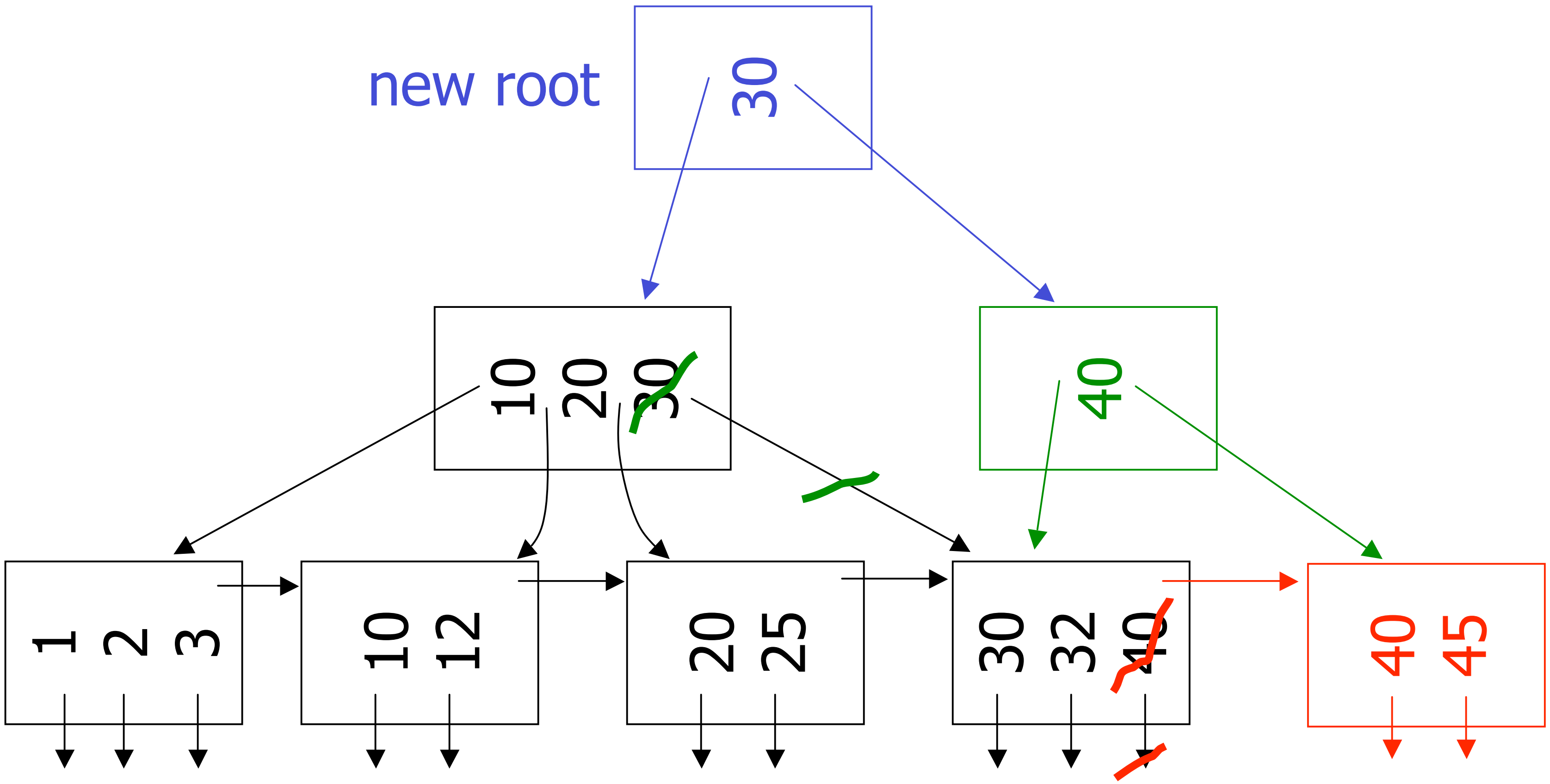
(c) Insert key = 160

n=3



(d) New root, insert 45

n=3



Deletion from B^+-tree

(a) Simple case - no example

(b) Coalesce with neighbour (sibling)

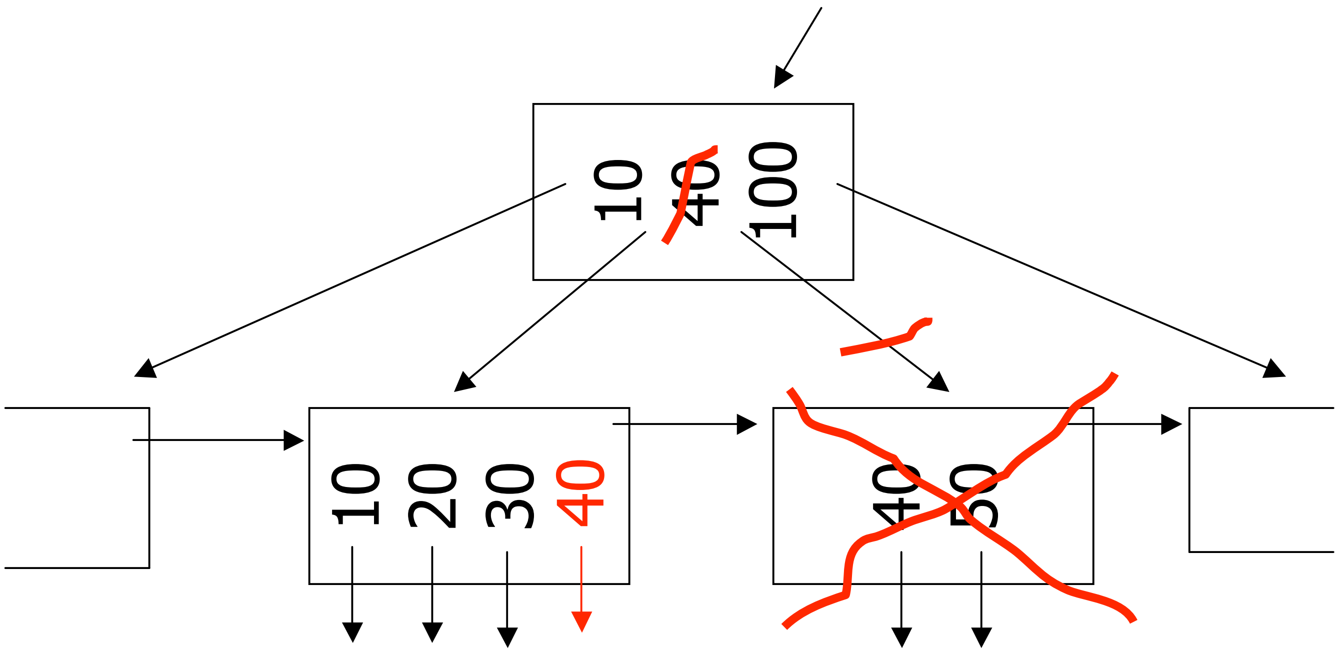
(c) Re-distribute keys

(d) Cases (b) or (c) at non-leaf

(b) Coalesce with sibling

- Delete 50

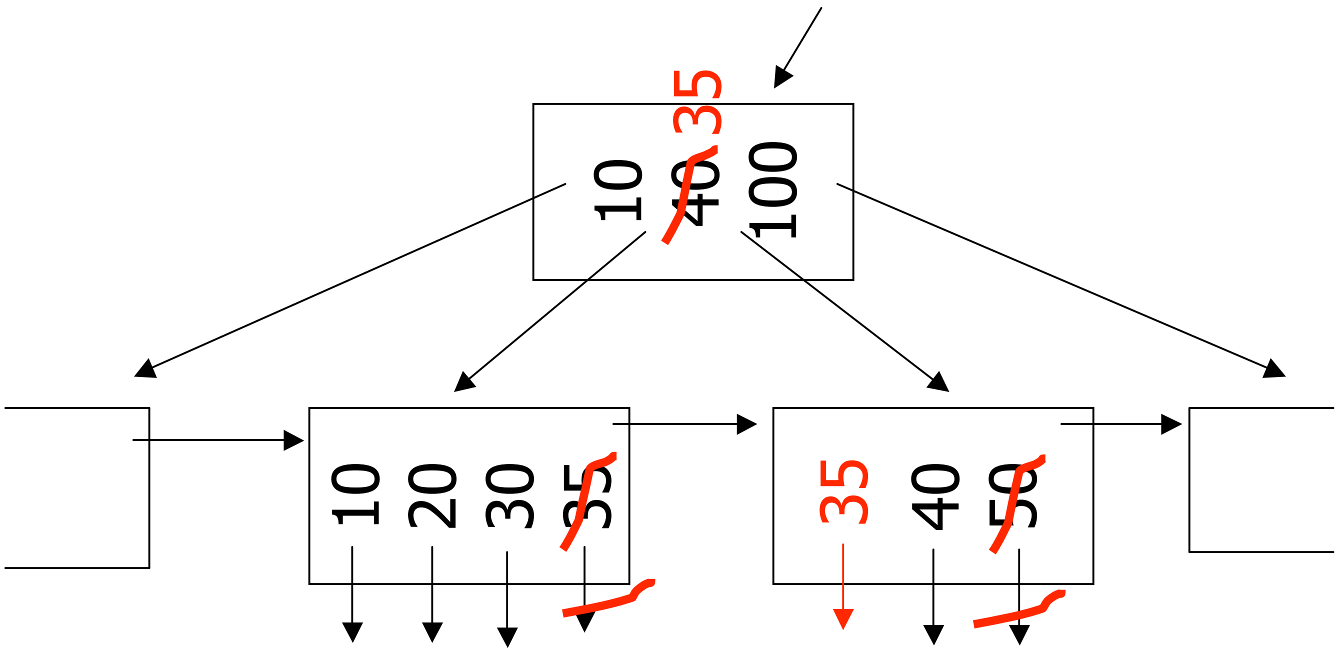
$\boxed{n=4}$



(c) Redistribute keys

- Delete 50

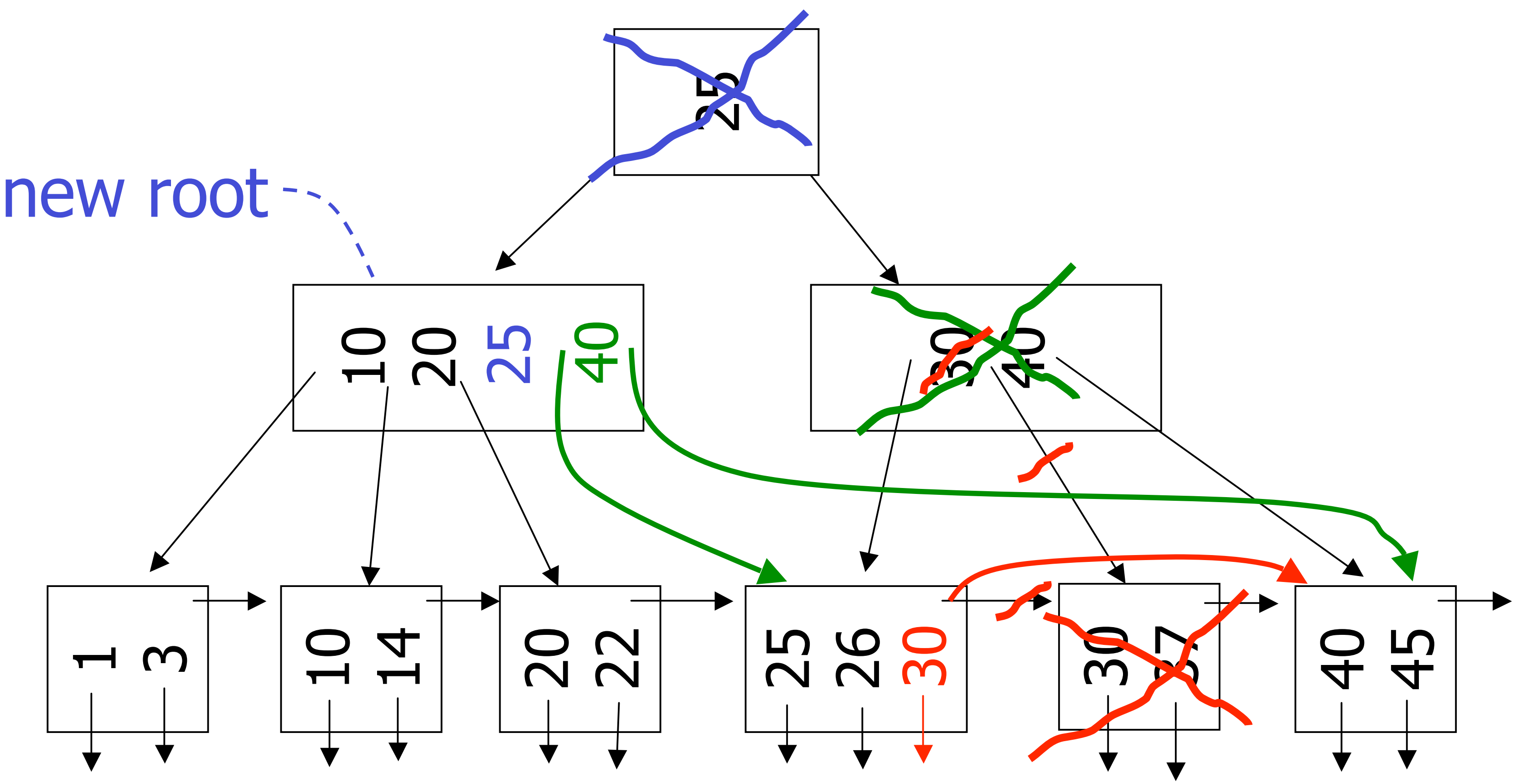
n=4



(d) Non-leaf coalesce

- Delete 37

n=4



Alternative B$^+$-tree deletion

- In practice, coalescing is often **not** implemented (hard, and often not worth it)
- An alternative is to use tombstones.
- Periodic **global rebuilding** may be used to remove tombstones when they start taking too much space.

Problem session: Analysis of B^+-trees

- What is the height of a B^+-tree with **N** leaves and room for **n** pointers in a node?
- What is the worst case I/O cost of
  - Searching?
  - Inserting and deleting?

B^+-tree summary

- Height ≤ $\mathbf{1+log_{n/2}\ N}$, typically 3 or 4.
- Best search time we could hope for!
  (To be shown in exercises.)
- If keeping top node(s) in memory, the number of I/Os can be reduced.
- Updates: Same cost as search, except for **rebalancing**.

Problem session

- Consider problem 2 on the hand-out, which asks for a proof that B-trees are optimal among pointer-based indexes.

Sorting using B-trees

- In internal memory, sorting can be done in O(n log n) time by inserting the keys into a balanced search tree.
- The number of I/Os for sorting by inserting into a B-tree is $O(N \log_B N)$.
- This is more than a factor B slower than multiway mergesort.

Next: Buffering in B-trees

- Based on slides by Gerth Brodal, covering a paper published in 2004 at the SODA conference.

More on rebalancing

- The book claims (on page 645):
  "It will be a rare event that calls for splitting or merging of blocks".
- This is true (in particular at the top levels), but a little hard to see.
- Easier seen for **weight-balanced B-trees**.

Weight-balanced B-trees

(based on [Pagh03], where n corresponds to B/2)

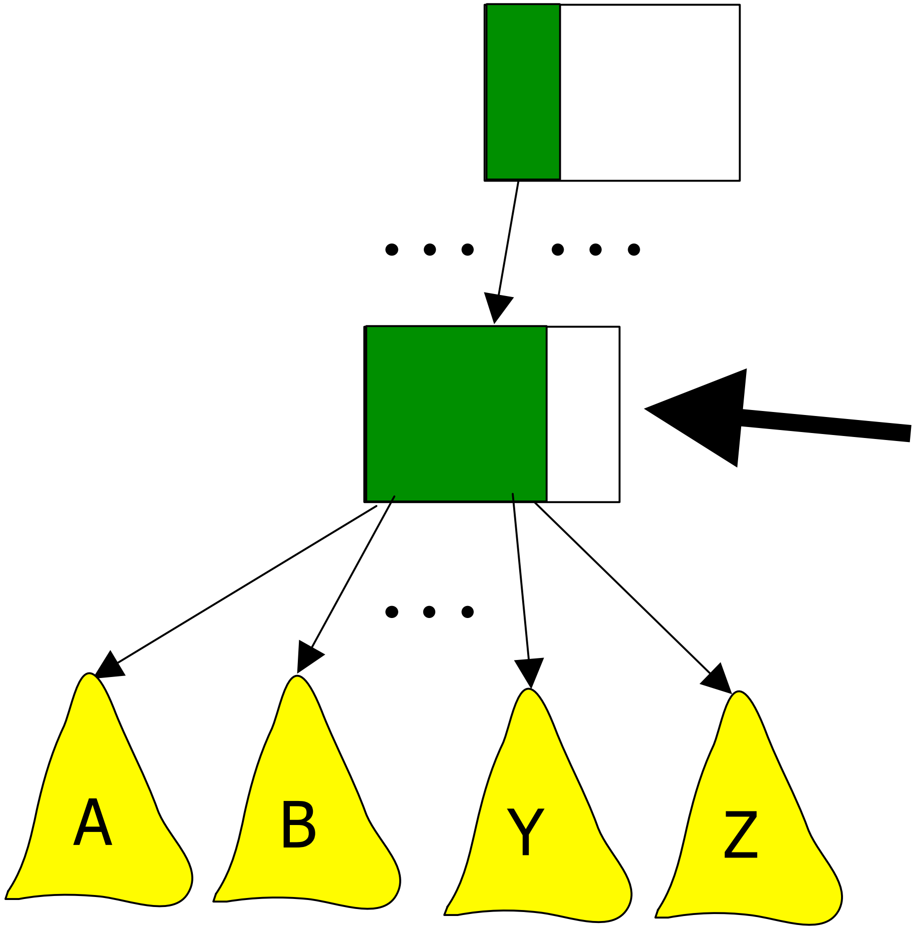
- Remove the **B^+-tree** invariant:
  There must be $\lfloor \mathbf{(n+1)/2} \rfloor$ non-null pointers in a node.
- Add new **weight** invariant:
  A node at height **i** must have weight (number of leaves in the subtree below) that is between $\mathbf{(n/4)^i}$ and $\mathbf{4(n/4)^i}$. (Again, the root is an exception.)

Weight-balanced B-trees

Consequences of the **weight** invariant:

- Tree height is $\leq 1+\log_{n/4} N$ (almost same)
- A node at height i with weight, e.g., $2(n/4)^i$ will not need rebalancing until there have been at least $(n/4)^i$ updates in its subtree. **(Why?)**

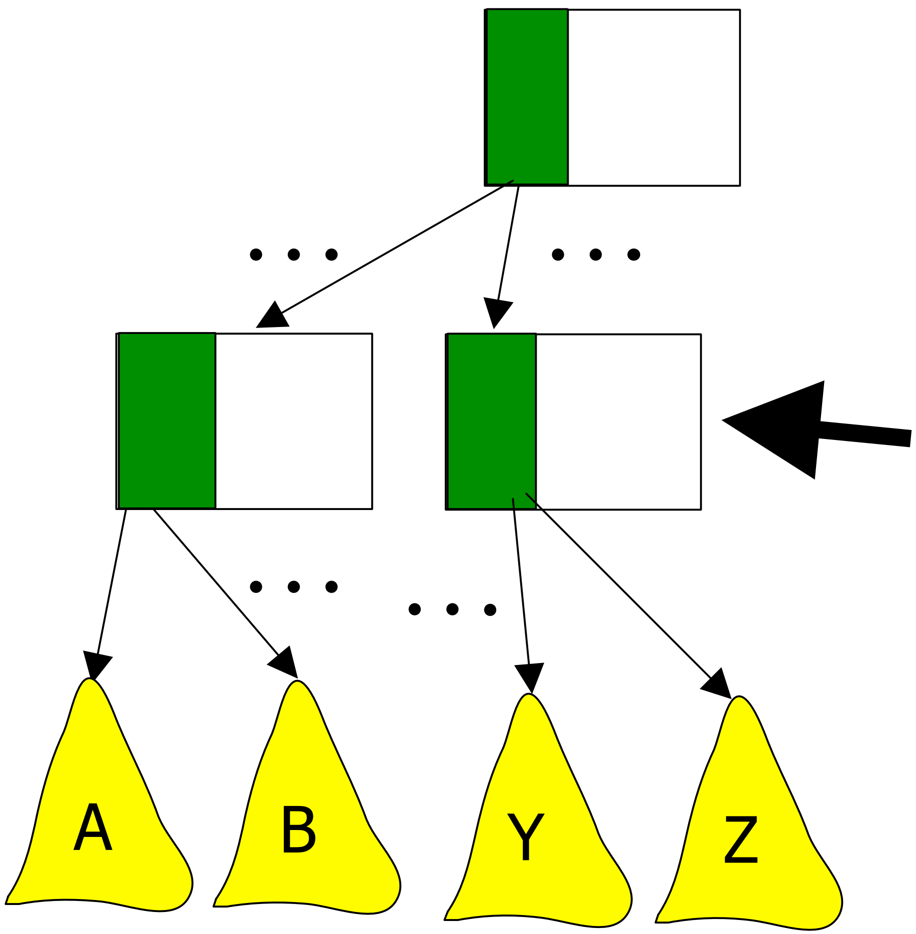
Rebalancing weight



More than **4(n/4)** ^{i} leaves in subtree $\Rightarrow$ weight balance invariant violated

New insertion in subtree

Rebalancing weight



Node is split into two nodes of weight around **$2(n/4)^i$**, i.e., **far** from violating the invariant (details in [Pagh03])

Weight-balanced B-trees

Summary of properties

- Deletions similar to insertions (or: use tombstones and global rebuilding).
- Search in time $\mathbf{O(\log_n N)}$.
- A node at height $\mathbf{i}$ is rebalanced (costing $\mathbf{O(1)}$ I/Os) once for every $\mathbf{\Omega((n/4)^i)}$ updates in its subtree.

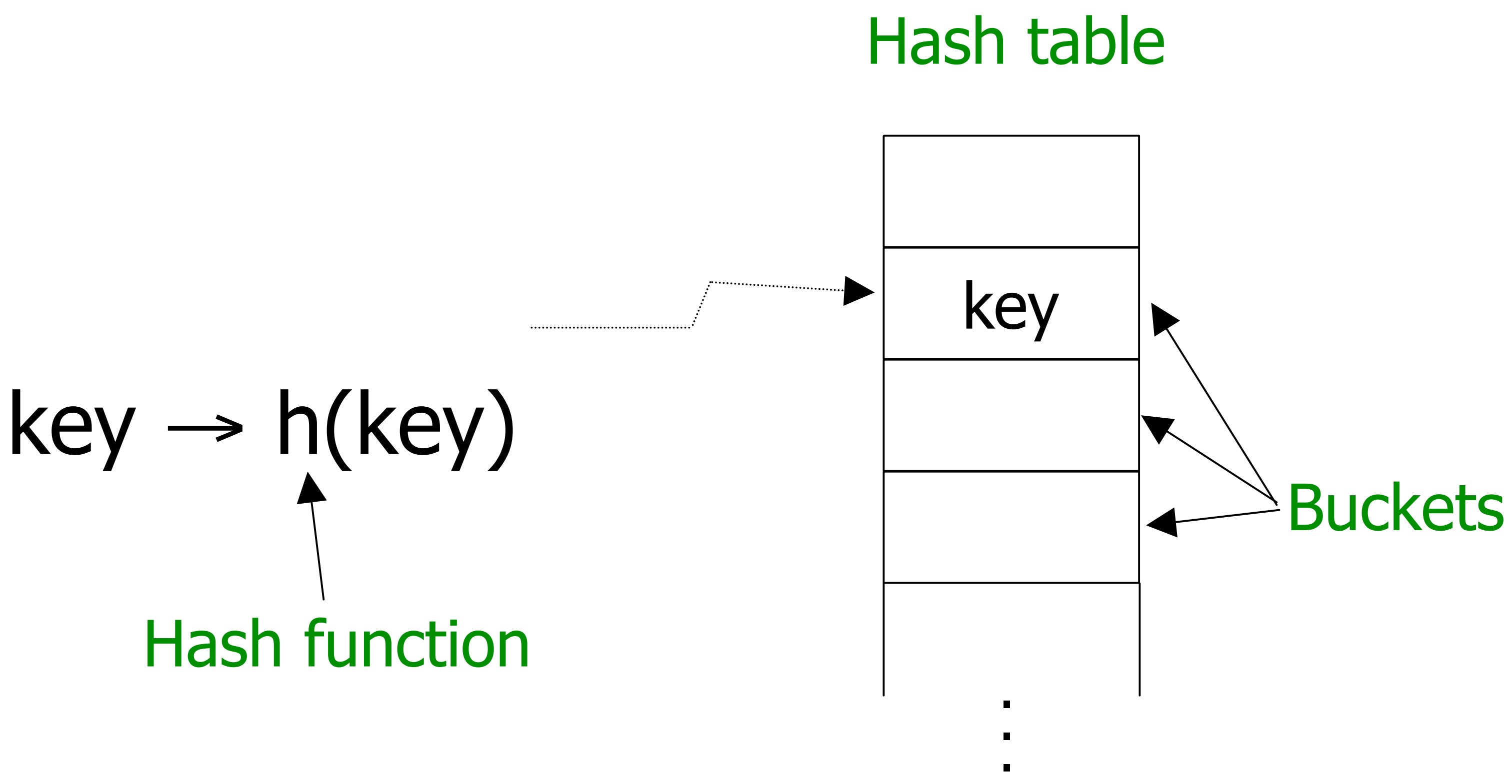
Other kinds of B-trees

- **String B-trees**: Fast searches even if keys span many blocks. (April 4 lecture.)
- **Persistent B-trees**: Make searches in any previous version of the tree, e.g. ”find x at time t”. The time for a search is $O(\log_B N)$, where N is the **total** number of keys inserted in the tree. (March 21 lecture.)

Next: Hash indexes

- You may recall that in internal memory, **hashing** can be used to quickly locate a specific key.
- The same technique can be used on external memory.
- However, advantage over search trees is smaller than internally.

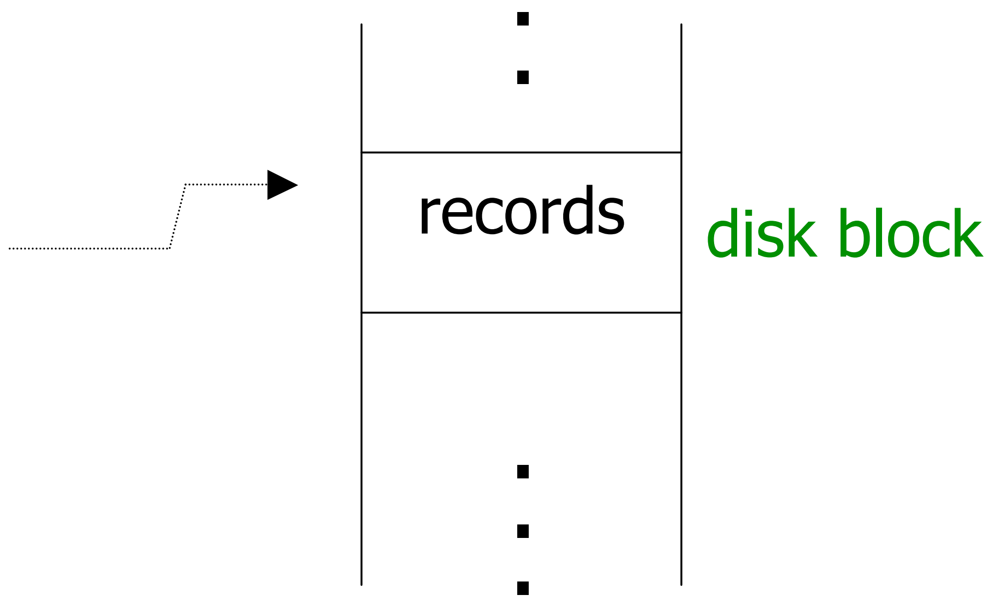
Hashing in a nutshell



Typical implementation of buckets: Linked lists

Hashing as primary index

$\text{key} \rightarrow h(\text{key})$

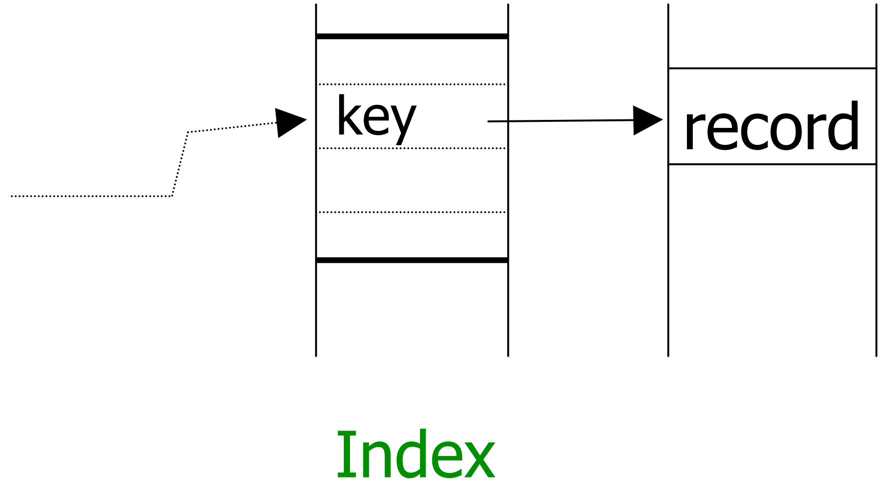


Note on terminology:

The word "indexing" is often used synonymously with "B-tree indexing".

Hashing as secondary index

key → h(key)



Today we discuss hashing as **primary index**.
Can always be transformed to a secondary index using indirection, as above.

Choosing a hash function

Book's suggestions (p. 650):

- Key = $x_1 \ x_2 \ldots x_n$, n byte character string: $h(Key) = (x_1 + x_2 + \_.. \ x_n)$ **mod** b
- Key is an integer: $h(Key) = Key$ **mod** b

PROBLEM:
For any fixed function, there are key sets that make it behave very badly (Why?)

Choosing a randomized function

Another approach (not mentioned in book):

- Choose h **at random** from some set of functions.
- This can make the hashing scheme behave well **regardless** of the key set.
- E.g., "universal hashing" makes chained hashing perform well (in theory and practice).
- Details out of scope for this course...

Insertions and overflows

INSERT:

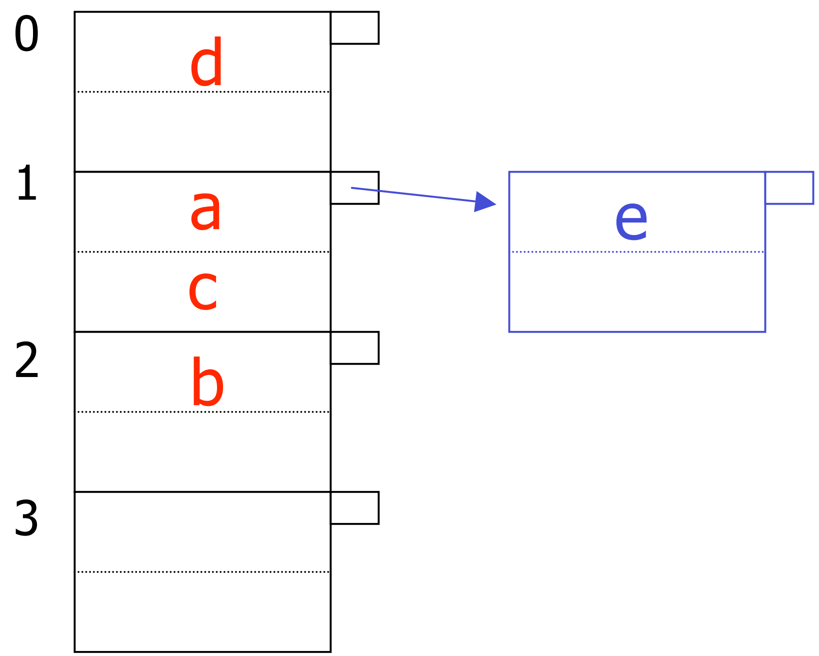
h(a) = 1

h(b) = 2

h(c) = 1

h(d) = 0

h(e) = 1



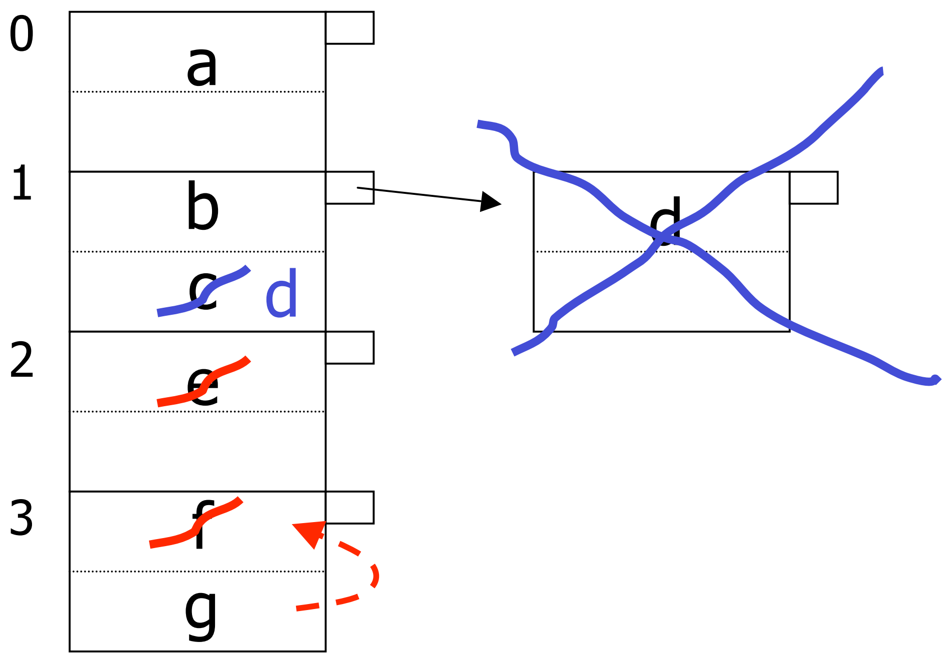
Deletions

Delete:

e

f

c



Analysis - external chained hashing (assuming truly random hash functions)

- N keys inserted, each block (bucket) in the hash table can hold B keys.
- Suppose the hash table has size $N/\alpha B$, i.e., "is a fraction α full".
- Expected number of overflow blocks: $(1-\alpha)^{-2} \cdot 2^{-\Omega(B)} N$ (proof omitted!)
- Good to have many keys in each bucket (an advantage of secondary indexes).

Sometimes life is easy...

- If B is sufficiently large compared to N, all overflow blocks can be kept in internal memory.
- Lookup in 1 I/O.
- Update in 2 I/Os.

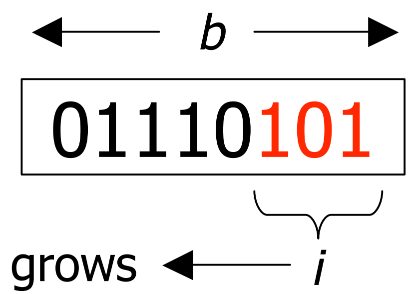
Coping with growth

- Overflows and global rebuilding
- Dynamic hashing
  - Extendible hashing
  - Linear hashing (next)
  - Uniform rehashing

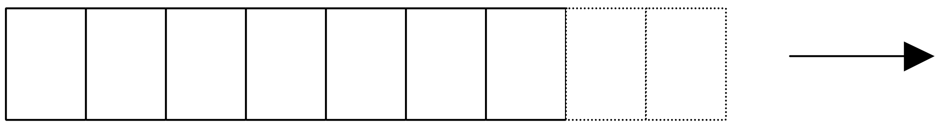
Linear hashing

Two ideas:

(a) Use ***i*** (low order) bits of h(K)

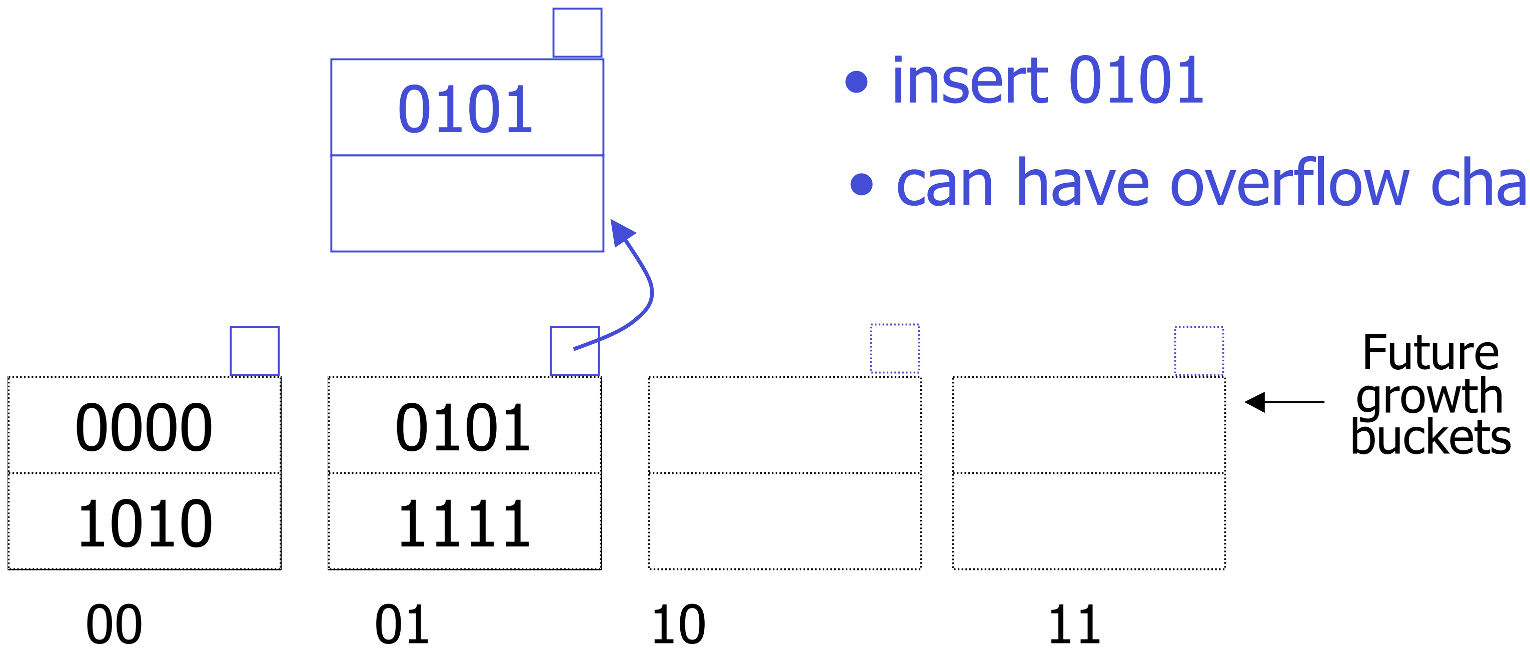


(b) Hash table grows one bucket at a time



Linear hashing example

b=4 bits, i=2, 2 keys/bucket



- insert 0101
- can have overflow chains!

m = 01 (max used block)

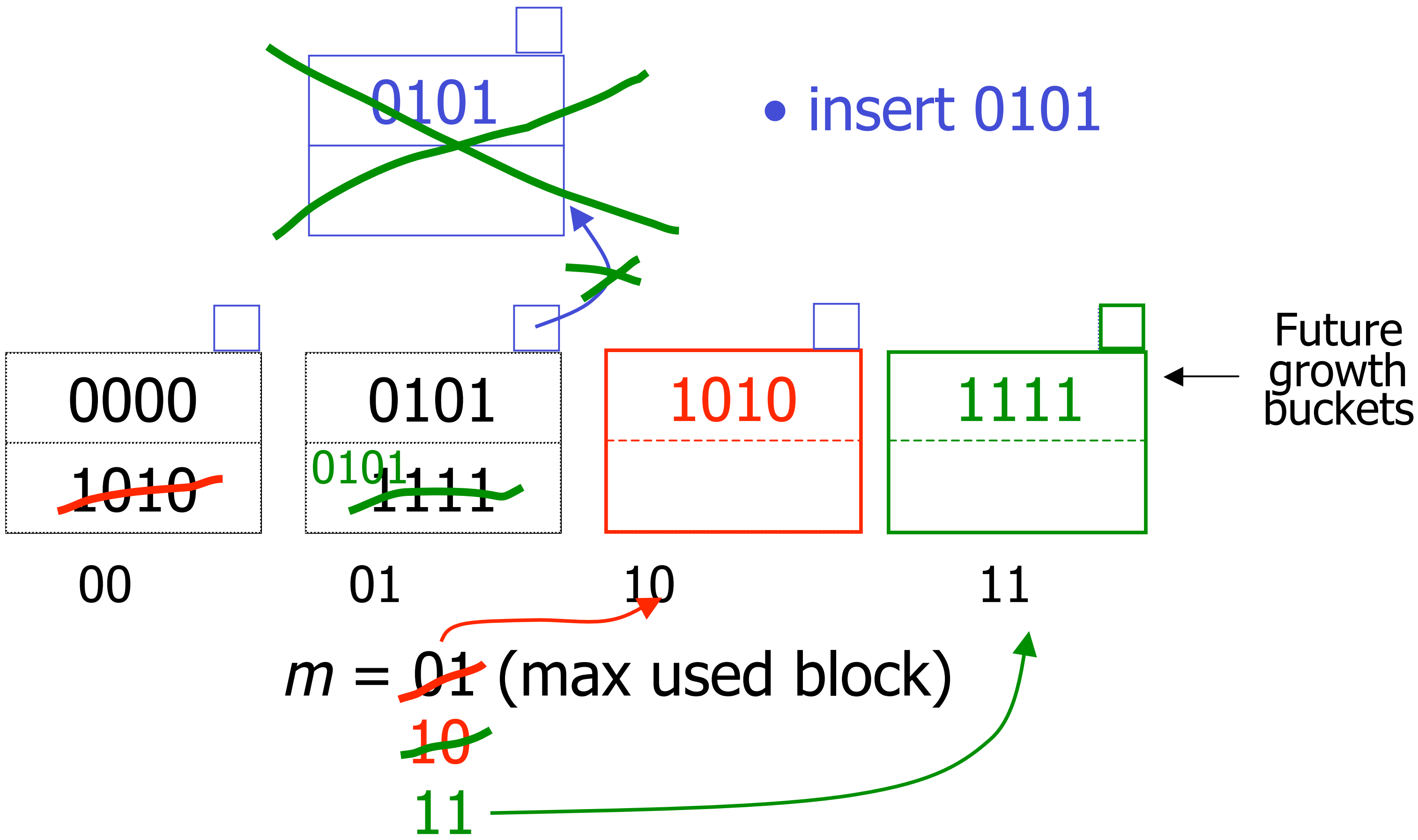
If *h(K)[i] ≤ m:* Look at bucket h(k)[i]

Otherwise: Look at bucket h(k)[i] -2^{i-1}

Linear hashing example, cont.

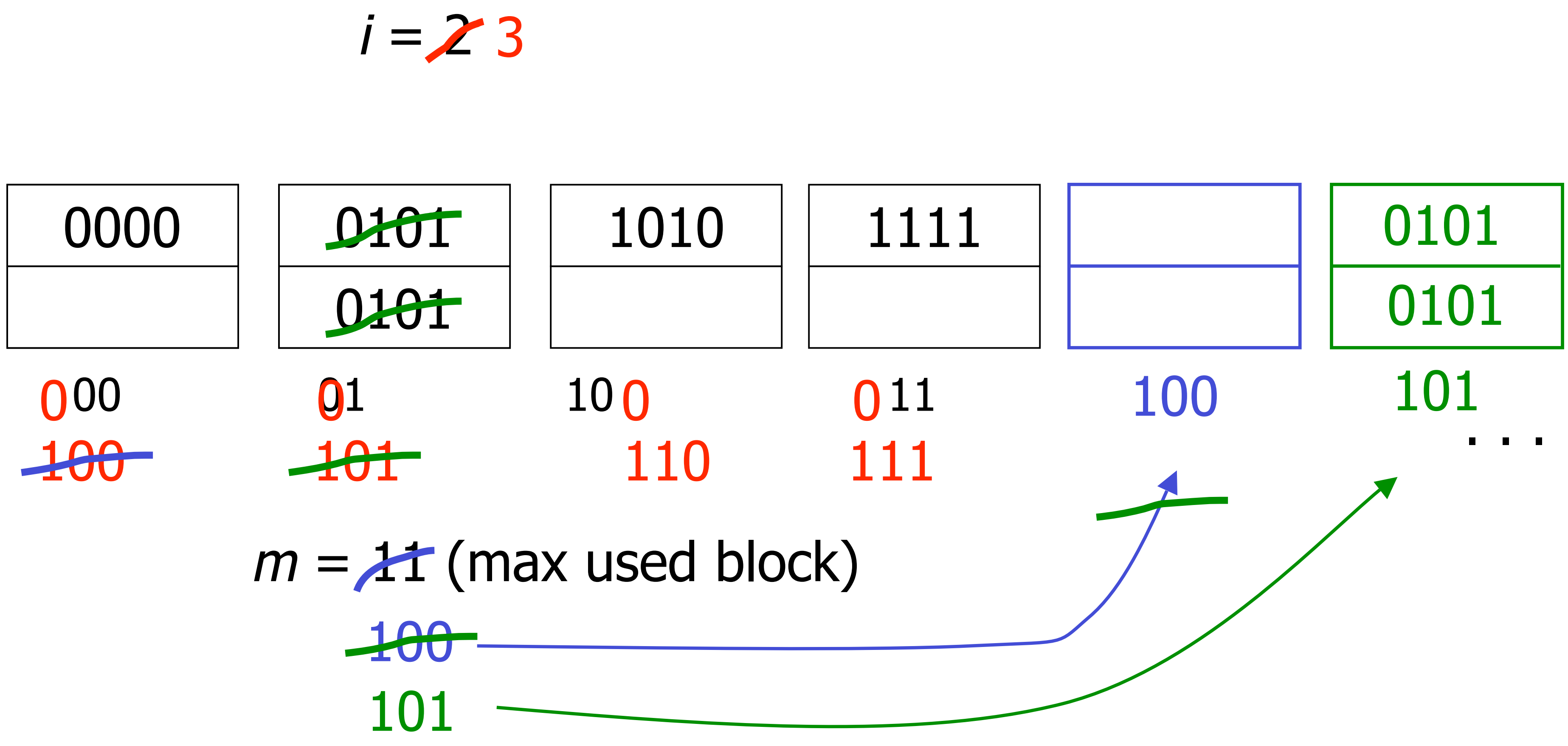
b=4 bits, i=2, 2 keys/bucket

- insert 0101



Linear hashing example, cont.

b=4 bits, i=2, 2 keys/bucket



When to expand the hash table?

- Keep track of the fraction $\alpha = (N/B)/m$
- If too close to 1 (e.g. $\alpha > 0.8$), increase m.
- Conversely, if α is too small, the table should be shrunk.

Performance of linear hashing

- Avoids using an index, lookup often 1 I/O.
- No good **worst-case** bound on lookups.
- Keys not placed uniformly in the table, so **worse** performance than in regular chained hashing.
- Extensions of linear hashing improve uniformity.

B-tree vs hash indexes

- Hashing good to search given key, e.g.,
  SELECT * FROM R WHERE A = 5
- Indexing (using B-trees) good for range searches, e.g.:
  SELECT * FROM R WHERE A > 5
- More applications to come...

Hashing and range searching

- ***Claim in book (p. 652):***
  "Hash tables do not support range queries"
- True, but they can be **used** to answer range queries in **O(1+Z/B)** I/Os, where Z is the number of results. (Alstrup, Brodal, Rauhe, 2001; Mortensen, Pagh, Patrascu 2005)
- Theoretical result, out of scope for ADBT.

Summary I

- **Indexing** is a "key" database technology.
- **Conventional indexes** (when few updates).
- **B-trees** (and variants) are more flexible
  - The choice of most DBMSs
    - Range queries.
    - Deterministic/reliable.
  - Theoretically “optimal”: $O(\log_B N)$ I/Os per operation.
  - Buffering can be used to achieve fast updates, at the cost of increasing height of the tree.

Summary II

- **External hash tables** support lookup of keys and updates in O(1) I/Os, expected - randomized algorithm.
- The actual constant (typically 1, 2, or 3) is a major concern (compare to B-trees).
- New ITU research: Close to 1 I/O per operation.
- <u>Growth management:</u> Linear hashing (extendible hashing, uniform rehashing).