1 Robust sort-merge join

Suppose we have relations $R_1(A_1, A_2, A_3)$ and $R_2(A_2, A_4)$, where $A_2$ is the primary key of $R_2$. All attributes have fixed length. The length of $A_1$ is 10 bytes, the length of $A_2$ is 20 bytes, the length of $A_3$ is 30 bytes, and the length of $A_4$ is 40 bytes. The attribute $A_2$ is a key for $R_2$. The relations reside on a disk with block size $B=3000$ bytes. We want to perform a sort-merge join of $R_1$ and $R_2$, using 101 blocks (i.e., 303000 bytes) of internal memory, of which one is reserved to be used as an output buffer.

In the following we consider a sort-merge join of $R_1$ and $R_2$, as described in RG. The replacement sort method that produces longer runs on average is not used, but we use the refinement that produces the join result as the runs of $R_1$ and $R_2$ are read concurrently from disk.

**a)** For what values of $|R_1|$ and $|R_2|$ does the sort-merge join use two passes? (Write a condition involving $|R_1|$ and $|R_2|$.)

**b)** Assume that $|R_1| = 10,000$ and $|R_2| = 30,000$. How many I/Os are used for the sort-merge join in the worst case? Count also I/Os to write the output.

It is pointed out in RG that the sort-merge join algorithm presented there may be doing a lot of work in the “atypical” case where the set of tuples with a particular value on the join attribute(s) does not fit into internal memory. The aim of the following is to arrive at an algorithm that handles the general case efficiently. Let $B(R)$ and $B(S)$ denote the number of disk blocks of two relations $R$ and $S$, respectively.

**c)** Modify the two-pass sort-merge join described in RG to get a sort-merge join that, in the worst case, uses $3(B(R) + B(S)) + O(B(R \bowtie S))$ I/Os to compute the sort-merge join of two relations $R$ and $S$. You may assume that internal memory has space for approximately $2\sqrt{B(R) + B(S)}$ blocks. Argue that your algorithm has the desired complexity.
2 Sort-merge-based join on partly sorted input

Relations are often stored on disk sorted according to some attribute(s), for example relations organized in a clustered B$^+$-tree index. Also, results of some relational algebra operations are produced in sorted order. For simplicity we assume here that a relation is stored in a sequence of consecutive disk blocks (no “holes”).

We now consider a 2-pass sort-merge join, as described in RG section 14.4.2, in the special case where one of the relations is already sorted according to the join attribute(s). Let $R$ denote the unsorted relation, and $S$ denote the sorted relation. We denote by $B(R)$ and $B(S)$ the number of disk blocks occupied by $R$ and $S$, respectively, and by $M$ the number of disk blocks that fit in internal memory.

a) Which part of the 2-pass sort-merge join algorithm may be skipped in this special case? Argue that the resulting I/O complexity is $3B(R) + B(S)$, not counting I/Os to write the output.

b) Describe a modification of the 2-pass sort-merge join algorithm that uses $3B(R) + B(S)$ I/Os, not counting I/Os to write the output, and works in the above special case if $M > \sqrt{B(R)} + 2$. Argue for the the I/O complexity and memory requirements of the algorithm.