Advanced Models and Programs

Abstract syntax, interpretation, checking; lexing and parsing

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Plan for today

- Plan for the next six lectures
- Abstract syntax trees
- Interpretation of abstract syntax
- Checking of abstract syntax
- From text to abstract syntax
  1. Lexing: character stream to token stream
  2. Parsing: token stream to abstract syntax

The next six AMP lectures

- Mon 9: Abstract syntax, interpretation, lexing, parsing
- Wed 11: Parsing, grammar transformations, compilation, stack machines
- Mon 16: Micro-C and pointers
- Wed 18: Bytecode, abstract machines, real machines
- Mon 23: The Scheme programming language, program generation and multi-stage programs
- Wed 25: (*Runtime code generation in a spreadsheet implementation* ***or*** *Runtime bytecode generation)*

A little language of expressions

- Example expressions:

```
7
x
7 + 9 * 10
x + y * z
x * (y + z) > 42
(x > 42) * 10
```

- An expression evaluates to an integer

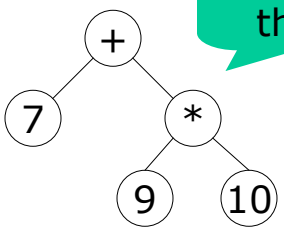
Abstract syntax trees

- An expression is a tree

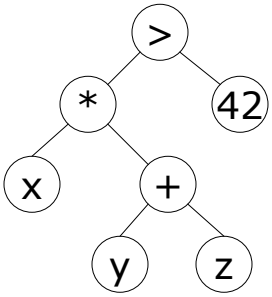
No paren-theses

```
7 + 9 * 10
```

```
7 + (9 * 10)
```

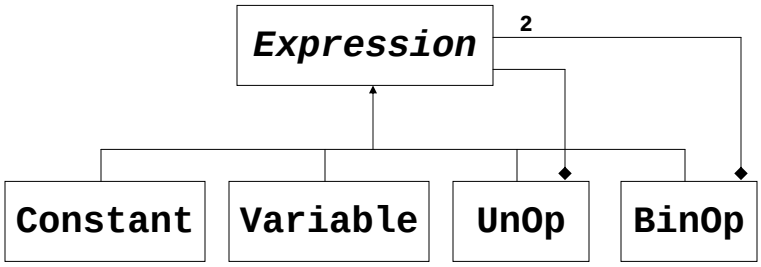



```
x * (y + z) > 42
```



Object-oriented representation of expressions

- A tree is an object structure
- Tree node kinds form a class hierarchy



C# classes for expressions

```
public abstract class Expression { }
public class Constant : Expression {
  private readonly int value;
}
public class Variable : Expression {
  private readonly String name;
}
public class BinOp : Expression {
  private readonly Operator op;
  private readonly Expression e1, e2;
}
public class UnOp : Expression {
  private readonly Operator op;
  private readonly Expression e1;
}
public enum Operator { Add, Sub, Mul, ... }
```

Example expressions

```
9 * 10
```

```
new BinOp(Operator.MUL,
          new Constant(9),
          new Constant(10))
```

In SML we would use a datatype

```
datatype expr =
   Constant of int
 | Variable of string
 | UnOp of operator * expr
 | BinOp of operator * expr * expr
datatype operator =
   Add | Sub | Mul | ...
```

- Expression representation is very similar

```
9 * 10
```

```
BinOp(MUL,
      Constant(9),
      Constant(10))
```


Evaluation of expressions, 1

```
public abstract class Expression {
  abstract public int Eval(Renv env);
}
public class Constant : Expression {
  private readonly int value;
  public override int Eval(REnv env) {
    return value;
  }
}
public class Variable : Expression {
  private readonly String name;
  public override int Eval(REnv env) {
    return env.GetVariable(name).value;
  }
}
```

Evaluation of expressions, 2

```
public class BinOp : Expression {
  ...
  public override int Eval(REnv env) {
    int v1 = e1.Eval(env);
    int v2 = e2.Eval(env);
    switch (op) {
    case Operator.Add:
      return v1 + v2;
    case Operator.Div:
      return v1 / v2;
    case Operator.Mul:
      return v1 * v2;
    ...
  }
}
```

The runtime environment maps variable to storage and value

```
public class REnv {
  private readonly Stack<Pair<String, Storage>> locals;
  public Storage GetVariable(String name) {
    ...
  }
}
public class Storage {
  public int value = 0;
}
```

- The storage can hold an int

```
env.GetVariable("x").value = 42;
```

Set value

```
return env.GetVariable("x").value;
```

Get value

The runtime environment is a stack of bindings

- This works well with nested scopes

```
let x = 2 in let y = x+3 in y*4 end end
```

[("x",2)]

[("y",5),("x",2)]

```
public class REnv {
  private readonly Stack<Pair<String, Storage>> locals;
  public Storage GetVariable(String name) {
    foreach (Pair<String, Storage> variable in locals)
      if (variable.Fst == name)
        return variable.Snd;
    throw new Exception("Unbound variable: " + name);
  }
}
```

Typed expressions

- We want to prohibit **`(4 > 3) * 10`**
- Distinguish types **`bool`** and **`int`**
- And an error type, for **`(4 > 3) * 10`**

```
abstract public class Type {
  public static readonly Type intType
    = new PrimitiveType("int");
  public static readonly Type boolType
    = new PrimitiveType("bool");
  public static readonly Type errorType
    = new PrimitiveType("*ERROR*");
}
public class PrimitiveType : Type {
  public readonly String name;
}
```

Type checking of expressions

Result is a Type

```
public abstract class Expression {
  abstract public int Eval(REnv env);
  abstract public Type Check(TEnv env);
}
public class Constant : Expression {
  private readonly int value;
  private readonly Type type;
  public override Type Check(TEnv env) {
    return type;
  }
}
public class Variable : Expression {
  public override Type Check(TEnv env) {
    return env.GetVariable(name);
  }
}
```

New

New

Type environ-ment

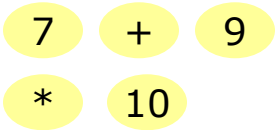
Type checking of binary operators

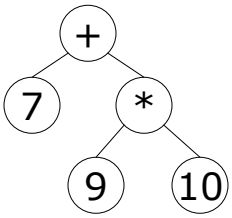
```
public class BinOp : Expression {
  public override Type Check(TEnv env) {
    Type t1 = e1.Check(env);
    Type t2 = e2.Check(env);
    switch (op) {
      case Operator.Add: case Operator.Div: ...
        if (t1 == Type.intType
            && t2 == Type.intType)
          return Type.intType;
        else
          throw new TypeException(...);
      case Operator.Eq: case Operator.Ge: ...
        if (t1 == Type.intType
            && t2 == Type.intType)
          return Type.boolType;
        else
          throw new TypeException();
  ...
} }
```

From text file to abstract syntax

- Programmers write text (source code), not abstract syntax

"7+9* 10"





Regular expressions (R.E.)

- A regular expression describes a set of strings

R.E. r	Meaning	Language L(r)
a	symbol a	{ "a" }
ε	empty sequence	{ "" }
$r_1\ r_2$	r_1 followed by r_2	$\{ s_1s_2 \mid s_1 \in L(r_1), s_2 \in L(r_2)\}$
r *	zero or more r	$\{ s_1 \ldots s_n \mid s_i \in L(r), n \geq 0\}$
$r_1 \mid r_2$	r_1 or else r_2	$L(r_1) \cup L(r_2)$

- ab* represents { "a", "ab", "abb", ... }
- (ab)* represents { "", "ab", "abab", ... }
- (a|b)* represents { "", "a", "b", "ab", "ba" ... }
- (a|b)c* represents ?

Regular expression abbreviations

Abbrev	Meaning	Expansion
[aeiou]	set	a | e | i | o | u
[0-9]	range	0 | 1 | … | 9
[a-zA-Z]	ranges	a | … | z | A | … | Z
r?	zero or one r	ε | r
r+	one or more r	r r*

- Alternative syntax (used in Coco/R):
  "aeiou" instead of "[aeiou]"
  "{ r }" instead of "r*"

Examples

- Non-negative integer constants
- Integer constants
- Java variable names
- Java floating-point constants
- Pre-1990 Danish car license plates
- Internet domains (e.g. www.pol.dk)
- Email addresses

Lexers and lexer generators

- A *lexer* transforms a character stream into a token stream
- A *lexer generator* takes as input a lexer specification, and generates a lexer
- A *lexer specification* is a collection of regular expressions

- Coco/R is a simple combined lexer generator and parser generator

Example lexer specification (expression language, Coco/R)

```
CHARACTERS
  letter = "ABCDEFGHIJKLMNOPQRSTUVWXYZabc...xyz".
  digit = "0123456789".
  cr  = '\r'.
  lf  = '\n'.
  tab = '\t'.

TOKENS
  ident  = letter {letter | digit}.
  number = digit {digit}.

IGNORE cr + lf + tab
```

From regular expression to finite automaton

- For every regular expression r there is a finite automaton that recognizes exactly the strings described by r
- The converse is also true
- Construction:

  Regular expression

  => Nondeterministic finite automaton (NFA)

  => Deterministic finite automaton (DFA)
- This gives a very efficient way of determining whether a string is described by a regular expression

Parsers and parser generators

- A parser transforms a token stream into an abstract syntax tree
- A parser generators turns a parser specification into a parser
- A parser specification is a grammer equipped with semantic actions, that is, instructions for AST building

Grammar for expressions

```
Expr ::=
    n                                  // constant
  | x                                  // variable
  | UnOper Expr
  | Expr BinOper Expr
  | ( Expr )
UnOper ::= - | !
BinOper ::= + | - | * | /
  | == | != | < | <= | > | >=
```

Start symbol

Terminal symbol, token

Nonterminal symbol

Rule

Example parser specification

```
Expr<out Expression e>     (. Expression e1, e2; Operator op; .)
= SimExpr<out e1>          (. e = e1; .)
  [ RelOp<out op>
    SimExpr<out e2>        (. e = new BinOp(op, e, e2); .)
  ]
.
SimExpr<out Expression e> (. Expression e1, e2; Operator op; .)
= Term<out e1>             (. e = e1; .)
  { AddOp<out op>
    Term<out e2>           (. e = new BinOp(op, e, e2); .)
  }
.
RelOp<out Operator op>      (. op = Operator.Bad; .)
= ( "=="                    (. op = Operator.Eq;  .)
  | "!="                    (. op = Operator.Ne;  .)
  | "<"                     (. op = Operator.Lt;  .)
  | "<="                    (. op = Operator.Le;  .)
  | ">"                     (. op = Operator.Gt;  .)
  | ">="                    (. op = Operator.Ge;  .)
  )
. // more rules
```

Grammar classes (Chomsky hierarchy, 1956)

- Type 3: Regular grammars; same expressiveness as regular expressions

  $A \rightarrow cB$ $A \rightarrow B$ $A \rightarrow c$ $A \rightarrow \varepsilon$
- Type 2: Context-free grammars (CFG)

  $A \rightarrow cBd$
- Type 1: Context-sensitive grammars, non-abbreviating rules

  $aAb \rightarrow acAdb$
- Type 0: Unrestricted grammars; same as term rewrite systems

  $0Ay \rightarrow 0$

The language of a grammar

- L(G) = { strings derivable from G }
  = { s | S → s }
  where S is the start symbol of G
- Parsing is inverse derivation:
  - Could this string be derived from S?
  - And if so, using which rules?
- There are CFG parsers, but they're slow
- Subclasses of CFG can be parsed fast:
  - LL, top-down parsers: Coco/R, ANTLR, ...
  - LR, bottom-up parsers: Yacc, Bison, Javacup, ... (Knuth 1968)

Wednesday

- Lecture:
  - Grammars often must be rewritten to be used as parser specifications
  - Stack machines
  - Compilation of expressions

- Exercises for Wednesday:
  - Extend the expression language
  - Extend the lexer and parser specifications
  - Extend the interpreter, checker and compiler