Note on Separately Matching Place and Link Graphs

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February 2005

We work within pure, concrete bigraphical reactive systems. Recall that a match of a redex R within an agent A comprises a context C, a wiring id_Z and a parameter d such that

$$A = C \circ (R \otimes \mathsf{id}_Z) \circ d.$$

Because composition of bigraphs is defined on place and link graphs independently, we can factor a match into its place and link graph constituents,

$$A_P = C_P \circ R_P \circ d \qquad \text{and} \qquad A_L = C_L \circ (R_L \otimes \mathsf{id}_Z) \circ d.$$

Thus, we can speak of place graph matches and link graph matches. We will say that a place and link graph match are *compatible* if, combined, they form a match.

Theorem 1. *Consider and agent A and a redex R, and assume R_L epi. Then a place-graph match of R_P within A_P has (up to isos) at most one compatible link-graph match.*

This theorem justifies implementations of bigraphical reactive systems that divide match-finding into two phases: first locate a place-graph match, then construct one compatible link-graph match.

Proof. Suppose both $(C_L, \mathsf{id}_Z, d_L)$ and $(C'_L, \mathsf{id}_{Z'}, d'_L)$ are compatible matches. Then

$$C_L \circ (R_L \otimes \mathsf{id}_Z) \circ d_L = A_L = C'_L \circ (R_L \otimes \mathsf{id}_{Z'}) \circ d'_L. \tag{1}$$

Both (d_P, d_L) and (d_P, d'_L) are discrete bigraphs, so neither d_L nor d'_L have any edges. Hence (d_P, d_L) and (d_P, d'_L) have identical support, as $|d_L| = |d'_L| = |d_P|$. Discreteness also implies that the link maps of d_L and d'_L are both one-one, so there exists some iso $\alpha : Z' \to Z$ with

$$d_L = \alpha \circ d'_L.$$

Thus $C_L \circ (R_L \otimes \mathsf{id}_Z) \circ d_L = C_L \circ (R_L \otimes \mathsf{id}_Z) \circ \alpha \circ d'_L$. But d'_L is discrete, hence epi, so by (1) we have

$$C_L \circ (R_L \otimes \mathsf{id}_Z) \circ \alpha = C'_L \circ (R_L \otimes \mathsf{id}_{Z'}).$$

Because R_L is the link-graph of a pure redex, it has the empty inner face. We invoke the tensor symmetries.

$$\begin{aligned} C_L \circ (R_L \otimes \mathsf{id}_Z) \circ \alpha &= C_L \circ (R_L \otimes \mathsf{id}_Z) \circ (\mathsf{id}_\epsilon \otimes \alpha) \\ &= C_L \circ \big((R_L \circ \mathsf{id}_\epsilon) \otimes (\mathsf{id}_Z \circ \alpha)\big) \\ &= C_L \circ \big((\mathsf{id}_{\mathrm{cod}(R_L)} \circ R_L) \otimes (\alpha \circ \mathsf{id}_{Z'})\big) \\ &= C_L \circ (\mathsf{id}_{\mathrm{cod}(R_L)} \otimes \alpha) \circ (R_L \otimes \mathsf{id}_{Z'}) \end{aligned}$$

A link graph is epi iff it has no idle names, so tensors preserve epis; in particular, $R_L \otimes \mathsf{id}_{Z'}$ is epi. Taking β to be the iso $\mathsf{id}_{\mathrm{cod}(R_L)} \otimes \alpha$, we find

$$C_L \circ \beta = C'_L.$$

By considering the support of d_L and d'_L, we see that $|Z| = |Z'|$; so id_Z and $\mathsf{id}_{Z'}$ are also isomorphic. Hence, if a link-graph match exists, it is determined uniquely up to iso by the place-graph match.

□

If we work out the interfaces of α and β, we see that the only freedom we have is to choose the set of names Z, with which we connect the parameter to the context. We prove now that this choice have no bearing on the generated reactions.

Corollary 2. *Consider an agent A and a reaction rule (R, R', ϱ), assume R_L epi, and consider some place-graph match of R_L within A_L. Then every compatible link-graph match yield support-equivalent reactions.*

Proof. Consider two link-graph matches (C_L, id_Z, d) and $(C'_L, \mathsf{id}_{Z'}, d')$ compatible with the given place-graph match. By Theorem 1, there exists isomorphisms α and β such that $d = \alpha \circ d'$ and $C'_L = C_L \circ \beta$; from the proof of Theorem 1 it is evident that $\beta = (\mathsf{id}_{\mathrm{cod}(R_L)} \otimes \alpha)$. Because R_L and R'_L have identical codomains, $\beta = (\mathsf{id}_{\mathrm{cod}(R'_L)} \otimes \alpha)$. Finally, by [1, Proposition 9.19] wiring commutes with instantiation, so $\varrho(d) = \varrho(\alpha \circ d') \asymp \alpha \circ \varrho(d')$, where $\asymp$ denotes support equivalence. We compute:

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\begin{aligned}
C_L \circ (R'_L \otimes \mathsf{id}_Z) \circ \varrho(d) &\asymp C'_L \circ \beta^{-1} \circ (R'_L \otimes \mathsf{id}_Z) \circ \alpha \circ \varrho(d') \\
&= C'_L \circ \beta^{-1} \circ (R'_L \otimes \mathsf{id}_Z) \circ (\mathsf{id}_\epsilon \otimes \alpha) \circ \varrho(d') \\
&= C'_L \circ \beta^{-1} \circ (R'_L \circ \mathsf{id}_\epsilon) \otimes (\mathsf{id}_Z \circ \alpha) \circ \varrho(d') \\
&= C'_L \circ \beta^{-1} \circ (\mathsf{id}_{\mathrm{cod}(R'_L)} \circ R'_L) \otimes (\alpha \circ \mathsf{id}_{Z'}) \circ \varrho(d') \\
&= C'_L \circ \beta^{-1} \circ (\mathsf{id}_{\mathrm{cod}(R'_L)} \otimes \alpha) \circ (R'_L \otimes \mathsf{id}_{Z'}) \circ \varrho(d') \\
&= C'_L \circ (\mathsf{id}_{\mathrm{cod}(R'_L)} \otimes \alpha)^{-1} \circ (\mathsf{id}_{\mathrm{cod}(R'_L)} \otimes \alpha) \circ (R'_L \otimes \mathsf{id}_{Z'}) \circ \varrho(d') \\
&= C'_L \circ (R'_L \otimes \mathsf{id}_{Z'}) \circ \varrho(d')
\end{aligned}
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References

[1] Ole Høgh Jensen and Robin Milner. Bigraphs and mobile processes (revised). Technical Report UCAM-CL-TR-580, University of Cambridge Computer Laboratory, February 2004.