The Definition of Standard ML

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Preface

A precise description of a programming language is a prerequisite for its implementation and for its use. The description can take many forms, each suited to a different purpose. A common form is a reference manual, which is usually a careful narrative description of the meaning of each construction in the language, often backed up with a formal presentation of the grammar (for example, in Backus-Naur form). This gives the programmer enough understanding for many of his purposes. But it is ill-suited for use by an implementer, or by someone who wants to formulate laws for equivalence of programs, or by a programmer who wants to design programs with mathematical rigour.

This document is a formal description of both the grammar and the meaning of a language which is both designed for large projects and widely used. As such, it aims to serve the whole community of people seriously concerned with the language. At a time when it is increasingly understood that programs must withstand rigorous analysis, particular for systems where safety is critical, a rigorous language presentation is even important for negotiators and contractors; for a robust program written in an insecure language is like a house built upon sand.

Most people have not looked at a rigorous language presentation before. To help them particularly, but also to put the present work in perspective for those more theoretically prepared, it will be useful here to say something about three things: the nature of Standard ML, the task of language definition in general, and the form of the present Definition.

Standard ML

Standard ML is a functional programming language, in the sense that the full power of mathematical functions is present. But it grew in response to a particular programming task, for which it was equipped also with full imperative power, and a sophisticated exception mechanism. It has an advanced form of parametric modules, aimed at organised development of large programs. Finally it is strongly typed, and it was the first language to provide a particular form of polymorphic type which makes the strong typing remarkably flexible. This combination of ingredients has not made it unduly large, but their novelty has been a fascinating challenge to semantic method (of which we say more below).

ML has evolved over fourteen years as a fusion of many ideas from many people. This evolution is described in some detail in Appendix E of the book, where also we acknowledge all those who have contributed to it, both in design and in implementation.

‘ML’ stands for meta language; this is the term logicians use for a language in which other (formal or informal) languages are discussed and analysed. Originally ML was conceived as a medium for finding and performing proofs in a logical language. Conducting rigorous argument as dialogue between person and machine
has been a strong research interest at Edinburgh and elsewhere, throughout these fourteen years. The difficulties are enormous, and make stern demands upon the programming language which is used for this dialogue. Those who are not familiar with computer-assisted reasoning may be surprised that a programming language, which was designed for this rather esoteric activity, should ever lay claim to being generally useful. On reflection, they should not be surprised. LISP is a prime example of a language invented for esoteric purposes and becoming widely used. LISP was invented for use in artificial intelligence (AI); the important thing about AI here is not that it is esoteric, but that it is difficult and varied; so much so, that anything which works well for it must work well for many other applications too.

The same can be said about the initial purpose of ML, but with a different emphasis. Rigorous proofs are complex things, which need varied and sophisticated presentation – particularly on the screen in interactive mode. Furthermore the proof methods, or strategies, involved are some of the most complex algorithms which we know. This all applies equally to AI, but one demand is made more strongly by proof than perhaps by any other application: the demand for rigour.

This demand established the character of ML. In order to be sure that, when the user and the computer claim to have together performed a rigorous argument, their claim is justified, it was seen that the language must be strongly typed. On the other hand, to be useful in a difficult application, the type system had to be rather flexible, and permit the machine to guide the user rather than impose a burden upon him. A reasonable solution was found, in which the machine helps the user significantly by inferring his types for him. Thereby the machine also confers complete reliability on his programs, in this sense: If a program claims that a certain result follows from the rules of reasoning which the user has supplied, then the claim may be fully trusted.

The principle of inferring useful structural information about programs is also represented, at the level of program modules, by the inference of signatures. Signatures describe the interfaces between modules, and are vital for robust large-scale programs. When the user combines modules, the signature discipline prevents him from mismatching their interfaces. By programming with interfaces and parametric modules, it becomes possible to focus on the structure of a large system, and to compile parts of it in isolation from one another – even when the system is incomplete.

This emphasis on types and signatures has had a profound effect on the language Definition. Over half this document is devoted to inferring types and signatures for programs. But the method used is exactly the same as for inferring what values a program delivers; indeed, a type or signature is the result of a kind of abstract evaluation of a program phrase.

In designing ML, the interplay among three activities – language design, definition and implementation – was extremely close. This was particularly true for the newest part, the parametric modules. This part of the language grew from
an initial proposal by David MacQueen, itself highly developed; but both formal
definition and implementation had a strong influence on the detailed design. In
general, those who took part in the three activities cannot now imagine how they
could have been properly done separately.

Language Definition

Every programming language presents its own conceptual view of computation.
This view is usually indicated by the names used for the phrase classes of the
language, or by its keywords: terms like package, module, structure, exception,
channel, type, procedure, reference, sharing, .... These terms also have their
abstract counterparts, which may be called semantic objects; these are what people
really have in mind when they use the language, or discuss it, or think in it. Also,
it is these objects, not the syntax, which represent the particular conceptual view
of each language; they are the character of the language. Therefore a definition of
the language must be in terms of these objects.

As is commonly done in programming language semantics, we shall loosely
talk of these semantic objects as meanings. Of course, it is perfectly possible to
understand the semantic theory of a language, and yet be unable to to understand
the meaning of a particular program, in the sense of its intention or purpose. The
aim of a language definition is not to formalise everything which could possibly
be called the meaning of a program, but to establish a theory of semantic objects
upon which the understanding of particular programs may rest.

The job of a language-definer is twofold. First – as we have already suggested
– he must create a world of meanings appropriate for the language, and must find
a way of saying what these meanings precisely are. Here, he meets a problem;
notation of some kind must be used to denote and describe these meanings – but
not a programming language notation, unless he is passing the buck and defining
one programming language in terms of another. Given a concern for rigour,
mathematical notation is an obvious choice. Moreover, it is not enough just to
write down mathematical definitions. The world of meanings only becomes mean-
ningful if the objects possess nice properties, which make them tractable. So the
language-definer really has to develop a small theory of his meanings, in the same
way that a mathematician develops a theory. Typically, after initially defining
some objects, the mathematician goes on to verify properties which indicate that
they are objects worth studying. It is this part, a kind of scene-setting, which
the language-definer shares with the mathematician. Of course he can take many
objects and their theories directly from mathematics, such as functions, relations,
trees, sequences, .... But he must also give some special theory for the objects
which make his language particular, as we do for types, structures and signatures
in this book; otherwise his language definition may be formal but will give no
insight.

The second part of the definer's job is to define evaluation precisely. This
means that he must define at least what meaning, \( M \), results from evaluating any phrase \( P \) of his language (though he need not explain exactly how the meaning results; that is he need not give the full detail of every computation). This part of his job must be formal to some extent, if only because the phrases \( P \) of his language are indeed formal objects. But there is another reason for formality. The task is complex and error-prone, and therefore demands a high level of explicit organisation (which is, largely, the meaning of ‘formality’); moreover, it will be used to specify an equally complex, error-prone and formal construction: an implementation.

We shall now explain the keystone of our semantic method. First, we need a slight but important refinement. A phrase \( P \) is never evaluated in vacuo to a meaning \( M \), but always against a background; this background – call it \( B \) – is itself a semantic object, being a distillation of the meanings preserved from evaluation of earlier phrases (typically variable declarations, procedure declarations, etc.). In fact evaluation is background-dependent – \( M \) depends upon \( B \) as well as upon \( P \).

The keystone of the method, then, is a certain kind of assertion about evaluation; it takes the form

\[
B \vdash P \Rightarrow M
\]

and may be pronounced: ‘Against the background \( B \), the phrase \( P \) evaluates to the meaning \( M \). The formal purpose of this Definition is no more, and no less, than to decree exactly which assertions of this form are true. This could be achieved in many ways. We have chosen to do it in a structured way, as others have, by giving rules which allow assertions about a compound phrase \( P \) to be inferred from assertions about its constituent phrases \( P_1, \ldots, P_n \).

The form of the Definition\(^1\)

We have written the Definition in a form suggested by the previous remarks. That is, we have defined our semantic objects in mathematical notation which is completely independent of Standard ML, and we have developed just enough of their theory to give sense to our rules of evaluation. Following another suggestion above, we have factored our task by describing abstract evaluation – the inference and checking of types and signatures (which can be done at compile-time) – completely separately from concrete evaluation. It really is a factorisation, because a full value in all its glory – you can think of it as a concrete object with a type attached – never has to be presented.

The resulting document is, we hope, valuable as the essential point of reference for Standard ML. If it is to play this role well, it must be supplemented by other

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\(^1\)The Definition has evolved through a sequence of three previous versions, circulated as Technical Reports. For those who have followed the sequence, we should point out that the treatment of equality types and of admissibility has been slightly modified in this publication to meet the claim for principal signatures. The changes are mainly in Sections 4.9, 5.5 and 5.13 and in the inference rules 19, 20, 29 and 65.
literature. Some expository books have already been written, and this Definition will be useful as a background reference for their readers. We have also become convinced, while writing the Definition, that we could not discuss many questions without making it far too long. Such questions are: Why were certain design choices made? What are their implications for programming? Was there a good alternative meaning for some constructs, or was our hand forced? What different forms of phrase are equivalent? What is the proof of certain claims? Many of these questions will not be answered by pedagogic texts either. So we are writing a Commentary on the Definition which will assist people in reading it, and which will serve as a bridge between the Definition and other texts.

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1 Introduction

This document formally defines Standard ML.

To understand the method of definition, at least in broad terms, it helps to consider how an implementation of ML is naturally organised. ML is an interactive language, and a program consists of a sequence of top-level declarations; the execution of each declaration modifies the top-level environment, which we call a basis, and reports the modification to the user.

In the execution of a declaration there are three phases: parsing, elaboration, and evaluation. Parsing determines the grammatical form of a declaration. Elaboration, the static phase, determines whether it is well-typed and well-formed in other ways, and records relevant type or form information in the basis. Finally evaluation, the dynamic phase, determines the value of the declaration and records relevant value information in the basis. Corresponding to these phases, our formal definition divides into three parts: grammatical rules, elaboration rules, and evaluation rules. Furthermore, the basis is divided into the static basis and the dynamic basis; for example, a variable which has been declared is associated with a type in the static basis and with a value in the dynamic basis.

In an implementation, the basis need not be so divided. But for the purpose of formal definition, it eases presentation and understanding to keep the static and dynamic parts of the basis separate. This is further justified by programming experience. A large proportion of errors in ML programs are discovered during elaboration, and identified as errors of type or form, so it follows that it is useful to perform the elaboration phase separately. In fact, elaboration without evaluation is just what is normally called compilation; once a declaration (or larger entity) is compiled one wishes to evaluate it – repeatedly – without re-elaboration, from which it follows that it is useful to perform the evaluation phase separately.

A further factoring of the formal definition is possible, because of the structure of the language. ML consists of a lower level called the Core language (or Core for short), a middle level concerned with programming-in-the-large called Modules, and a very small upper level called Programs. With the three phases described above, there is therefore a possibility of nine components in the complete language definition. We have allotted one section to each of these components, except that we have combined the parsing, elaboration and evaluation of Programs in one section. The scheme for the ensuing seven sections is therefore as follows:

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The Core provides many phrase classes, for programming convenience. But about half of these classes are derived forms, whose meaning can be given by translation into the other half which we call the Bare language. Thus each of
the three parts for the Core treats only the bare language; the derived forms are treated in Appendix A. This appendix also contains a few derived forms for Modules. A full grammar for the language is presented in Appendix B.

In Appendices C and D the initial basis is detailed. This basis, divided into its static and dynamic parts, contains the static and dynamic meanings of all predefined identifiers.

The semantics is presented in a form known as Natural Semantics. It consists of a set of rules allowing sentences of the form

\[ A \vdash \text{phrase} \Rightarrow A' \]

to be inferred, where \( A \) is often a basis (static or dynamic) and \( A' \) a semantic object – often a type in the static semantics and a value in the dynamic semantics. One should read such a sentence as follows: “against the background provided by \( A \), the phrase \( \text{phrase} \) elaborates – or evaluates – to the object \( A' \)”. Although the rules themselves are formal the semantic objects, particularly the static ones, are the subject of a mathematical theory which is presented in a succinct form in the relevant sections. This theory, particularly the theory of types and signatures, will benefit from a more pedagogic treatment in other publications; the treatment here is probably the minimum required to understand the meaning of the rules.

The robustness of the semantics depends upon theorems. Usually these have been proven, but the proof is not included. In two cases, however, they are presented as “claims” rather than theorems; these are the claim of principal environments in Section 4.12, and the claim of principal signatures in Section 5.13. We need further confirmation of our detailed proofs of these claims, before asserting them as theorems.
2 Syntax of the Core

2.1 Reserved Words

The following are the reserved words used in the Core. They may not (except = ) be used as identifiers.

abstype and andalso as case do datatype else
end exception fn fun handle if in infix
infixl let local nonfix of op open orelse
raise rec then type val with withtype while
( ) [ ] { } , : ; ... _ | = => -> #

2.2 Special constants

An integer constant is any non-empty sequence of digits, possibly preceded by a negation symbol (~). A real constant is an integer constant, possibly followed by a point (.) and one or more digits, possibly followed by an exponent symbol E and an integer constant; at least one of the optional parts must occur, hence no integer constant is a real constant. Examples: 0.7 3.32E5 3E-7 . Non-examples: 23 .3 4.5E1 1E2.0 .

We assume an underlying alphabet of 256 characters (numbered 0 to 255) such that the characters with numbers 0 to 127 coincide with the ASCII character set. A string constant is a sequence, between quotes ("), of zero or more printable characters (i.e., numbered 33–126), spaces or escape sequences. Each escape sequence starts with the escape character \, and stands for a character sequence. The escape sequences are:

\n A single character interpreted by the system as end-of-line.
\t Tab.
\^c The control character c, where c may be any character with number 64–95. The number of \^c is 64 less than the number of c.
\ddd The single character with number ddd (3 decimal digits denoting an integer in the interval [0,255]).
\" "
\\ This sequence is ignored, where f·f stands for a sequence of one or more formatting characters.

The formatting characters are a subset of the non-printable characters including at least space, tab, newline, formfeed. The last form allows long strings to be written on more than one line, by writing \ at the end of one line and at the start of the next.

We denote by SCon the class of special constants, i.e., the integer, real, and string constants; we shall use scon to range over SCon.
2.3 Comments

A comment is any character sequence within comment brackets (* *) in which
comment brackets are properly nested. An unmatched comment bracket should
be detected by the compiler.

2.4 Identifiers

The classes of identifiers for the Core are shown in Figure 1. We use var, tyvar to
range over Var, TyVar etc. For each class X marked “long” there is a class longX
of long identifiers; if x ranges over X then longx ranges over longX. The syntax of
these long identifiers is given by the following:

longx ::= x identifier
strid₁⋯stridₙ.x qualified identifier (n ≥ 1)

The qualified identifiers constitute a link between the Core and the Modules.
Throughout this document, the term “identifier”, occurring without an adjective,
refers to non-qualified identifiers only.

An identifier is either alphanumeric: any sequence of letters, digits, primes (')
and underbars (_,) starting with a letter or prime, or symbolic: any non-empty
sequence of the following symbols

! % & $ # + - / : < = > ? @ \ - ' ^ | *

In either case, however, reserved words are excluded. This means that for example
# and | are not identifiers, but ## and |= are identifiers. The only exception
to this rule is that the symbol =, which is a reserved word, is also allowed as
an identifier to stand for the equality predicate. The identifier = may not be
re-bound; this precludes any syntactic ambiguity.

A type variable tyvar may be any alphanumeric identifier starting with a prime;
the subclass EtyVar of TyVar, the equality type variables, consists of those which
start with two or more primes. The subclass ImpTyVar of TyVar, the imperative
type variables, consists of those which start with one or two primes followed by
an underbar. The complement AppTyVar = TyVar \ ImpTyVar consists of the
applicative type variables. The other six classes (Var, Con, ExCon, TyCon, Lab
and StrId) are represented by identifiers not starting with a prime. However, * is
excluded from TyCon, to avoid confusion with the derived form of tuple type (see Figure 22). The class Lab is extended to include the \textit{numeric} labels 1 2 3 \ldots, i.e. any numeral not starting with 0.

TyVar is therefore disjoint from the other six classes. Otherwise, the syntax class of an occurrence of identifier \textit{id} in a Core phrase (ignoring derived forms, Section 2.7) is determined thus:

1. Immediately before "." – i.e. in a long identifier – or in an \textit{open} declaration, \textit{id} is a structure identifier. The following rules assume that all occurrences of structure identifiers have been removed.

2. At the start of a component in a record type, record pattern or record expression, \textit{id} is a record label.

3. Elsewhere in types \textit{id} is a type constructor, and must be within the scope of the type binding or datatype binding which introduced it.

4. Elsewhere, \textit{id} is an exception constructor if it occurs in the scope of an exception binding which introduces it as such, or a value constructor if it occurs in the scope of a datatype binding which introduced it as such; otherwise it is a value variable.

It follows from the last rule that no value declaration can make a "hole" in the scope of a value or exception constructor by introducing the same identifier as a variable; this is because, in the scope of the declaration which introduces \textit{id} as a value or exception constructor, any occurrence of \textit{id} in a pattern is interpreted as the constructor and not as the binding occurrence of a new variable.

By means of the above rules a compiler can determine the class to which each identifier occurrence belongs; for the remainder of this document we shall therefore assume that the classes are all disjoint.

2.5 Lexical analysis

Each item of lexical analysis is either a reserved word, a numeric label, a special constant or a long identifier. Comments and formatting characters separate items (except within string constants; see Section 2.2) and are otherwise ignored. At each stage the longest next item is taken.

2.6 Infixed operators

An identifier may be given \textit{infix status} by the \texttt{infix} or \texttt{infixr} directive, which may occur as a declaration; this status only pertains to its use as a \texttt{var}, a \texttt{con} or an \texttt{excon} within the scope (see below) of the directive. (Note that qualified identifiers never have infix status.) If \textit{id} has infix status, then "\texttt{exp}_1 \texttt{id} \texttt{exp}_2" (resp. "\texttt{pat}_1 \texttt{id} \texttt{pat}_2") may occur – in parentheses if necessary – wherever the application
"id\{1=exp_1,2=exp_2\}" or its derived form "id(exp_1, exp_2)" (resp "id(pat_1, pat_2)") would otherwise occur. On the other hand, an occurrence of any long identifier (qualified or not) prefixed by op is treated as non-infixed. The only required use of op is in prefixing a non-infixed occurrence of an identifier id which has infix status; elsewhere, op, where permitted, has no effect. Infix status is cancelled by the nonfix directive. We refer to the three directives collectively as fixity directives.

The form of the fixity directives is as follows \((n \geq 1)\):

\[
\begin{align*}
\text{infix } & \langle d \rangle \ id_1 \cdots id_n \\
\text{infixr } & \langle d \rangle \ id_1 \cdots id_n \\
\text{nonfix } & id_1 \cdots id_n
\end{align*}
\]

where \(\langle d \rangle\) is an optional decimal digit \(d\) indicating binding precedence. A higher value of \(d\) indicates tighter binding; the default is 0. infix and infixr dictate left and right associativity respectively; association is always to the left for different operators of the same precedence. The precedence of infix operators relative to other expression and pattern constructions is given in Appendix B.

The scope of a fixity directive dir is the ensuing program text, except that if dir occurs in a declaration dec in either of the phrases

\[
\begin{align*}
\text{let } & \text{ dec in } \cdots \text{ end} \\
\text{local } & \text{ dec in } \cdots \text{ end}
\end{align*}
\]

then the scope of dir does not extend beyond the phrase. Further scope limitations are imposed for Modules.

These directives and op are omitted from the semantic rules, since they affect only parsing.

### 2.7 Derived Forms

There are many standard syntactic forms in ML whose meaning can be expressed in terms of a smaller number of syntactic forms, called the bare language. These derived forms, and their equivalent forms in the bare language, are given in Appendix A.

### 2.8 Grammar

The phrase classes for the Core are shown in Figure 2. We use the variable atexp to range over AtExp, etc.

The grammatical rules for the Core are shown in Figures 3 and 4.
AtExp       atomic expressions
ExpRow      expression rows
Exp         expressions
Match       matches
Mrule       match rules
Dec         declarations
ValBind     value bindings
TypBind     type bindings
DatBind     datatype bindings
ConBind     constructor bindings
ExBind      exception bindings
AtPat       atomic patterns
PatRow      pattern rows
Pat         patterns
Ty          type expressions
TyRow       type-expression rows

Figure 2: Core Phrase Classes

The following conventions are adopted in presenting the grammatical rules, and in their interpretation:

- The brackets \{\} enclose optional phrases.

- For any syntax class X (over which \(x\) ranges) we define the syntax class Xseq (over which \(xseq\) ranges) as follows:

  \[
  xseq ::= x \quad \text{(singleton sequence)}
  \]

  \[
  \quad (x_1, \ldots, x_n) \quad \text{(sequence, } n \geq 1) 
  \]

  (Note that the "\(...\)" used here, meaning syntactic iteration, must not be confused with "\ldots" which is a reserved word of the language.)

- Alternative forms for each phrase class are in order of decreasing precedence; this resolves ambiguity in parsing, as explained in Appendix B.

- L (resp. R) means left (resp. right) association.

- The syntax of types binds more tightly than that of expressions.

- Each iterated construct (e.g. \texttt{match}, \ldots) extends as far right as possible; thus, parentheses may be needed around an expression which terminates with a match, e.g. \texttt{"fn match"}, if this occurs within a larger match.
\[ \text{atexp} ::= \text{scon} \]
\[ \langle \text{op}\rangle \text{longvar} \]
\[ \langle \text{op}\rangle \text{longcon} \]
\[ \langle \text{op}\rangle \text{longexcon} \]
\[ \{ \langle \text{exprow} \rangle \} \]
\[ \text{let} \ \text{dec} \ \text{in} \ \text{exp} \ \text{end} \]
\[ ( \ \text{exp} \ ) \]
\[ \text{exprow} ::= \text{lab} = \text{exp} \ ( , \ \text{exprow} ) \]
\[ \text{exp} ::= \text{atexp} \]
\[ \text{exp} \ \text{atexp} \]
\[ \text{exp}_1 \ \text{id} \ \text{exp}_2 \]
\[ \text{exp} : \ \text{ty} \]
\[ \text{exp} \ \text{handle} \ \text{match} \]
\[ \text{raise} \ \text{exp} \]
\[ \text{fn} \ \text{match} \]
\[ \text{match} ::= \text{mrule} \ ( | \ \text{match} ) \]
\[ \text{mrule} ::= \text{pat} \ \Rightarrow \ \text{exp} \]
\[ \text{dec} ::= \text{val} \ \text{valbind} \]
\[ \text{type} \ \text{typbind} \]
\[ \text{datatype} \ \text{datbind} \]
\[ \text{abstype} \ \text{datbind} \ \text{with} \ \text{dec} \ \text{end} \]
\[ \text{exception} \ \text{exbind} \]
\[ \text{local} \ \text{dec}_1 \ \text{in} \ \text{dec}_2 \ \text{end} \]
\[ \text{open} \ \text{longstrid}_1 \ \cdots \ \text{longstrid}_n \]
\[ \text{dec}_1 \ \langle ; \rangle \ \text{dec}_2 \]
\[ \text{infix} \ \langle d \rangle \ \text{id}_1 \ \cdots \ \text{id}_n \]
\[ \text{infixr} \ \langle d \rangle \ \text{id}_1 \ \cdots \ \text{id}_n \]
\[ \text{nonfix} \ \text{id}_1 \ \cdots \ \text{id}_n \]
\[ \text{valbind} ::= \text{pat} = \text{exp} \ ( \text{and} \ \text{valbind} ) \]
\[ \text{rec} \ \text{valbind} \]
\[ \text{typbind} ::= \text{tyvarseq} \ \text{tycon} = \ \text{ty} \ ( \text{and} \ \text{typbind} ) \]
\[ \text{datbind} ::= \text{tyvarseq} \ \text{tycon} = \ \text{conbind} \ ( \text{and} \ \text{datbind} ) \]
\[ \text{conbind} ::= \langle \text{op}\rangle \text{con} \ \langle \text{of} \ \text{ty} \rangle \ ( | \ \text{conbind} ) \]
\[ \text{exbind} ::= \langle \text{op}\rangle \text{excon} \ \langle \text{of} \ \text{ty} \rangle \ ( \text{and} \ \text{exbind} ) \]
\[ \langle \text{op}\rangle \text{excon} = \langle \text{op}\rangle \text{longexcon} \ ( \text{and} \ \text{exbind} ) \]

Figure 3: Grammar: Expressions, Matches, Declarations and Bindings
2.9 Syntactic Restrictions

- No pattern may contain the same var twice. No expression row, pattern row or type row may bind the same lab twice.

- No binding valbind, typbind, datbind or exbind may bind the same identifier twice; this applies also to value constructors within a datbind.

- In the left side tyvarseq tycon of any typbind or datbind, tyvarseq must not contain the same tyvar twice. Any tyvar occurring within the right side must occur in tyvarseq.

- For each value binding pat = exp within rec, exp must be of the form fn match, possibly constrained by one or more type expressions. The derived form of function-value binding given in Appendix A, page 68, necessarily obeys this restriction.
3 Syntax of Modules

For Modules there are further reserved words, identifier classes and derived forms. There are no further special constants; comments and lexical analysis are as for the Core. The derived forms for modules concern functors and appear in Appendix A.

3.1 Reserved Words

The following are the additional reserved words used in Modules.

\[
\text{eqtype} \quad \text{functor} \quad \text{include} \quad \text{sharing} \\
\text{sig} \quad \text{signature} \quad \text{struct} \quad \text{structure}
\]

3.2 Identifiers

The additional syntax classes for Modules are SigId (signature identifiers) and FunId (functor identifiers); they may be either alphanumeric – not starting with a prime – or symbolic. The class of each identifier occurrence is determined by the grammatical rules which follow. Henceforth, therefore, we consider all identifier classes to be disjoint.

3.3 Infixed operators

In addition to the scope rules for fixity directives given for the Core syntax, there is a further scope limitation: if \( \text{dir} \) occurs in a structure-level declaration \( \text{strdec} \) in any of the phrases

\[
\text{let } \text{strdec} \text{ in } \cdots \text{ end} \\
\text{local } \text{strdec} \text{ in } \cdots \text{ end} \\
\text{struct } \text{strdec} \text{ end}
\]

then the scope of \( \text{dir} \) does not extend beyond the phrase.

One effect of this limitation is that fixity is local to a generative structure expression – in particular, to such an expression occurring as a functor body. A more liberal scheme (which is under consideration) would allow fixity directives to appear also as specifications, so that fixity may be dictated by a signature expression; furthermore, it would allow an \textit{open} or \textit{include} construction to restore the fixity which prevailed in the structures being opened, or in the signatures being included. This scheme is not adopted at present.

3.4 Grammar for Modules

The phrase classes for Modules are shown in Figure 5. We use the variable \( \text{strexp} \) to range over StrExp, etc. The conventions adopted in presenting the grammatical rules for Modules are the same as for the Core. The grammatical rules are shown
3.4 Grammar for Modules

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>StrExp</td>
<td>structure expressions</td>
</tr>
<tr>
<td>StrDec</td>
<td>structure-level declarations</td>
</tr>
<tr>
<td>StrBind</td>
<td>structure bindings</td>
</tr>
<tr>
<td>SigExp</td>
<td>signature expressions</td>
</tr>
<tr>
<td>SigDec</td>
<td>signature declarations</td>
</tr>
<tr>
<td>SigBind</td>
<td>signature bindings</td>
</tr>
<tr>
<td>Spec</td>
<td>specifications</td>
</tr>
<tr>
<td>ValDesc</td>
<td>value descriptions</td>
</tr>
<tr>
<td>TypDesc</td>
<td>type descriptions</td>
</tr>
<tr>
<td>DatDesc</td>
<td>datatype descriptions</td>
</tr>
<tr>
<td>ConDesc</td>
<td>constructor descriptions</td>
</tr>
<tr>
<td>ExDesc</td>
<td>exception descriptions</td>
</tr>
<tr>
<td>StrDesc</td>
<td>structure descriptions</td>
</tr>
<tr>
<td>SharEq</td>
<td>sharing equations</td>
</tr>
<tr>
<td>FunDec</td>
<td>functor declarations</td>
</tr>
<tr>
<td>FunBind</td>
<td>functor bindings</td>
</tr>
<tr>
<td>FunSigExp</td>
<td>functor signature expressions</td>
</tr>
<tr>
<td>FunSpec</td>
<td>functor specifications</td>
</tr>
<tr>
<td>FunDesc</td>
<td>functor descriptions</td>
</tr>
<tr>
<td>TopDec</td>
<td>top-level declarations</td>
</tr>
</tbody>
</table>

Figure 5: Modules Phrase Classes

In Figures 6, 7 and 8.

It should be noted that functor specifications (FunSpec) cannot occur in programs; neither can the associated functor descriptions (FunDesc) and functor signature expressions (FunSigExp). The purpose of a funspec is to specify the static attributes (i.e. functor signature) of one or more functors. This will be useful, in fact essential, for separate compilation of functors. If, for example, a functor \( g \) refers to another functor \( f \) then — in order to compile \( g \) in the absence of the declaration of \( f \) — at least the specification of \( f \) (i.e. its functor signature) must be available. At present there is no special grammatical form for a separately compilable “chunk” of text — which we may like to call a module — containing a fundec together with a funspec specifying its global references. However, below in the semantics for Modules it is defined when a declared functor matches a functor signature specified for it. This determines exactly those functor environments (containing declared functors such as \( f \)) into which the separately compiled “chunk” containing the declaration of \( g \) may be loaded.
\[
\begin{align*}
\text{strexp} & ::= \quad \text{struct strdec end} \quad \text{generative} \\
& \quad \text{longstrid} \quad \text{structure identifier} \\
& \quad \text{funid ( strexp )} \quad \text{functor application} \\
& \quad \text{let strdec in strexp end} \quad \text{local declaration} \\
\text{strdec} & ::= \quad \text{dec} \quad \text{declaration} \\
& \quad \text{structure strbind} \quad \text{structure} \\
& \quad \text{local strdec}_1 \text{ in strdec}_2 \text{ end} \quad \text{local} \\
& \quad \text{empty} \\
& \quad \text{sequential} \\
\text{strbind} & ::= \quad \text{strid ( : sigexp ) = strexp ( and strbind )} \\
\text{sigexp} & ::= \quad \text{sig spec end} \quad \text{generative} \\
& \quad \text{sigid} \quad \text{signature identifier} \\
\text{sigdec} & ::= \quad \text{signature sigbind} \quad \text{single} \\
& \quad \text{empty} \\
& \quad \text{sequential} \\
\text{sigbind} & ::= \quad \text{sigid = sigexp ( and sigbind )} \\
\end{align*}
\]

Figure 6: Grammar: Structure and Signature Expressions

3.5 Syntactic Restrictions

- No binding \textit{strbind}, \textit{sigbind}, or \textit{funbind} may bind the same identifier twice.

- No description \textit{valdesc}, \textit{typdesc}, \textit{datdesc}, \textit{exdesc}, \textit{strdesc} or \textit{fundesc} may describe the same identifier twice; this applies also to value constructors within a \textit{datdesc}.
3.5 Syntactic Restrictions

\[ \text{spec} ::= \text{val } \text{valdesc} \quad \text{value} \\
\text{type } \text{tydesc} \quad \text{type} \\
\text{eqtype } \text{tydesc} \quad \text{eqtype} \\
\text{datatype } \text{datdesc} \quad \text{datatype} \\
\text{exception } \text{exdesc} \quad \text{exception} \\
\text{structure } \text{strdesc} \quad \text{structure} \\
\text{sharing } \text{shareq} \quad \text{sharing} \\
\text{local } \text{spec}_1 \text{ in } \text{spec}_2 \text{ end} \quad \text{local} \\
\text{open } \text{longstrid}_1 \cdots \text{longstrid}_n \quad \text{open } (n \geq 1) \\
\text{include } \text{sigid}_1 \cdots \text{sigid}_n \quad \text{include } (n \geq 1) \\
\text{empty shared} \\
\text{sequential} \\
\]
\[
\text{fundec} ::= \text{functor funbind} \quad \text{single}
\]
\[
\text{fundec}_1 ; \text{fundec} \quad \text{empty}
\]
\[
\text{funbind} ::= \text{funid} \ ( \text{strid} : \text{sigexp}) \ (\text{sigexp'}) = \text{strexp} \quad \text{functor binding (and funbind)}
\]
\[
\text{funsigexp} ::= ( \text{strid} : \text{sigexp}) : \text{sigexp'} \quad \text{functor signature expression}
\]
\[
\text{funspec} ::= \text{functor fundesc} \quad \text{functor specification}
\]
\[
\text{funspec} \_1 ; \text{funspec} \_2 \quad \text{empty}
\]
\[
\text{funspec} \_1 ; \text{funspec} \_2 \quad \text{sequence}
\]
\[
\text{fundesc} ::= \text{funid funsigexp} \ (\text{and fundesc})
\]
\[
\text{topdec} ::= \text{strdec} \quad \text{structure-level declaration}
\]
\[
\text{sigdec} \quad \text{signature declaration}
\]
\[
\text{fundec} \quad \text{functor declaration}
\]

\textbf{Note:} No \textit{topdec} may contain, as an initial segment, a shorter top-level declaration followed by a semicolon.

Figure 8: Grammar: Functors and Top-level Declarations

### 3.6 Closure Restrictions

The semantics presented in later sections requires no restriction on reference to non-local identifiers. For example, it allows a signature expression to refer to external signature identifiers and (via \textit{sharing} or \textit{open}) to external structure identifiers; it also allows a functor to refer to external identifiers of any kind.

However, implementers who want to provide a simple facility for separate compilation may want to impose the following restrictions (ignoring references to identifiers bound in the initial basis \(B_0\), which may occur anywhere):

1. In any signature binding \(\text{sigid} = \text{sigexp}\), the only non-local references in \(\text{sigexp}\) are to signature identifiers.

2. In any functor description \(\text{funid} \ (\text{strid} : \text{sigexp}) : \text{sigexp'}\), the only non-local references in \(\text{sigexp}\) and \(\text{sigexp'}\) are to signature identifiers, except that \(\text{sigexp'}\) may refer to \(\text{strid}\) and its components.

3. In any functor binding \(\text{funid} \ (\text{strid} : \text{sigexp}) \ (\text{: sigexp'}) = \text{strexp}\), the only non-local references in \(\text{sigexp}, \text{sigexp'}\) and \(\text{strexp}\) are to functor and signature identifiers, except that both \(\text{sigexp'}\) and \(\text{strexp}\) may refer to \(\text{strid}\) and its components.

In the last two cases the final qualification allows, for example, sharing constraints to be specified between functor argument and result. (For a completely precise
definition of these closure restrictions, see the comments to rules 66 (page 40), 91 (page 43) and 96 (page 43) in the static semantics of modules, Section 5.)

The significance of these restrictions is that they may ease separate compilation; this may be seen as follows. If one takes a module to be a sequence of signature declarations, functor specifications and functor declarations satisfying the above restrictions then the elaboration of a module can be made to depend on the initial static basis alone (in particular, it will not rely on structures outside the module). Moreover, the elaboration of a module cannot create new free structure or type names, so name consistency (as defined in Section 5.2, page 32) is automatically preserved across separately compiled modules. On the other hand, imposing these restrictions may force the programmer to write many more sharing equations than is needed if functors and signature expressions can refer to free structures.
4 Static Semantics for the Core

Our first task in presenting the semantics – whether for Core or Modules, static or dynamic – is to define the objects concerned. In addition to the class of syntactic objects, which we have already defined, there are classes of so-called semantic objects used to describe the meaning of the syntactic objects. Some classes contain simple semantic objects; such objects are usually identifiers or names of some kind. Other classes contain compound semantic objects, such as types or environments, which are constructed from component objects.

4.1 Simple Objects

All semantic objects in the static semantics of the entire language are built from identifiers and two further kinds of simple objects: type constructor names and structure names. Type constructor names are the values taken by type constructors; we shall usually refer to them briefly as type names, but they are to be clearly distinguished from type variables and type constructors. Structure names play an active role only in the Modules semantics; they enter the Core semantics only because they appear in structure environments, which (in turn) are needed in the Core semantics only to determine the values of long identifiers. The simple object classes, and the variables ranging over them, are shown in Figure 9. We have included TyVar in the table to make visible the use of \( \alpha \) in the semantics to range over TyVar.

\[
\begin{align*}
\alpha \text{ or } \text{tyvar} & \in \text{TyVar} \quad \text{type variables} \\
t & \in \text{TyName} \quad \text{type names} \\
m & \in \text{StrName} \quad \text{structure names}
\end{align*}
\]

Figure 9: Simple Semantic Objects

Each \( \alpha \in \text{TyVar} \) possesses a boolean equality attribute, which determines whether or not it admits equality, i.e. whether it is a member of EtyVar (defined on page 4). Independently hereof, each \( \alpha \) possesses a boolean attribute, the imperative attribute, which determines whether it is imperative, i.e. whether it is a member of ImpTyVar (defined on page 4) or not.

Each \( t \in \text{TyName} \) has an arity \( k \geq 0 \), and also possesses an equality attribute. We denote the class of type names with arity \( k \) by \( \text{TyName}^{(k)} \).

With each special constant \( scon \) we associate a type name \( \text{type}(scon) \) which is either \text{int}, \text{real} or \text{string} as indicated by Section 2.2.

4.2 Compound Objects

When \( A \) and \( B \) are sets \( \text{Fin} A \) denotes the set of finite subsets of \( A \), and \( A \rightarrow B \) denotes the set of finite maps (partial functions with finite domain) from \( A \) to \( B \).
4.2 Compound Objects

The domain and range of a finite map, \( f \), are denoted \( \text{Dom} \ f \) and \( \text{Ran} \ f \). A finite map will often be written explicitly in the form \( \{ a_1 \mapsto b_1, \ldots, a_k \mapsto b_k \} \), \( k \geq 0 \); in particular the empty map is \( \{ \} \). We shall use the form \( \{ x \mapsto e ; \phi \} \) – a form of set comprehension – to stand for the finite map \( f \) whose domain is the set of values \( x \) which satisfy the condition \( \phi \), and whose value on this domain is given by \( f(x) = e \).

When \( f \) and \( g \) are finite maps the map \( f + g \), called \( f \) modified by \( g \), is the finite map with domain \( \text{Dom} \ f \cup \text{Dom} \ g \) and values

\[
(f + g)(a) = \text{if } a \in \text{Dom} \ g \text{ then } g(a) \text{ else } f(a).
\]

The compound objects for the static semantics of the Core Language are shown in Figure 10. We take \( \cup \) to mean disjoint union over semantic object classes. We also understand all the defined object classes to be disjoint.

\[
\begin{align*}
\tau & \in \text{Type} = \text{TyVar} \cup \text{RecType} \cup \text{FunType} \cup \text{ConsType} \\
(\tau_1, \ldots, \tau_k) \text{ or } \tau^{(k)} & \in \text{Type}^k \\
(\alpha_1, \ldots, \alpha_k) \text{ or } \alpha^{(k)} & \in \text{TyVar}^k \\
g & \in \text{RecType} = \text{Lab} \xrightarrow{\text{fin}} \text{Type} \\
\tau \rightarrow \tau' & \in \text{FunType} = \text{Type} \times \text{Type} \\
\text{ConsType} & = \bigcup_{k \geq 0} \text{ConsType}^{(k)} \\
\tau^{(k)} t & \in \text{ConsType}^{(k)} = \text{Type}^k \times \text{TyName}^{(k)} \\
\theta \text{ or } \Lambda \alpha^{(k)} . \tau & \in \text{TypeFcn} = \bigcup_{k \geq 0} \text{TyVar}^k \times \text{Type} \\
\sigma \text{ or } \forall \alpha^{(k)} . \tau & \in \text{TypeScheme} = \bigcup_{k \geq 0} \text{TyVar}^k \times \text{Type} \\
S \text{ or } (m, E) & \in \text{Str} = \text{StrName} \times \text{Env} \\
(\theta, CE) & \in \text{TyStr} = \text{TypeFcn} \times \text{ConEnv} \\
SE & \in \text{StrEnv} = \text{StrId} \xrightarrow{\text{fin}} \text{Str} \\
TE & \in \text{TyEnv} = \text{TyCon} \xrightarrow{\text{fin}} \text{TyStr} \\
CE & \in \text{ConEnv} = \text{Con} \xrightarrow{\text{fin}} \text{TypeScheme} \\
VE & \in \text{VarEnv} = (\text{Var} \cup \text{Con} \cup \text{ExCon}) \xrightarrow{\text{fin}} \text{TypeScheme} \\
EE & \in \text{ExConEnv} = \text{ExCon} \xrightarrow{\text{fin}} \text{Type} \\
E \text{ or } (SE, TE, VE, EE) & \in \text{Env} = \text{StrEnv} \times \text{TyEnv} \times \text{VarEnv} \times \text{ExConEnv} \\
T & \in \text{TyNameSet} = \text{Fin}(\text{TyName}) \\
U & \in \text{TyVarSet} = \text{Fin}(\text{TyVar}) \\
C \text{ or } T, U, E & \in \text{Context} = \text{TyNameSet} \times \text{TyVarSet} \times \text{Env}
\end{align*}
\]

Figure 10: Compound Semantic Objects

Note that \( \Lambda \) and \( \forall \) bind type variables. For any semantic object \( A \), tynames \( A \) and tyvars \( A \) denote respectively the set of type names and the set of type variables occurring free in \( A \). Moreover, imptyvars \( A \) and apptyvars \( A \) denote respectively
the set of imperative type variables and the set of applicative type variables occurring free in $A$.

### 4.3 Projection, Injection and Modification

**Projection:** We often need to select components of tuples – for example, the variable-environment component of a context. In such cases we rely on variable names to indicate which component is selected. For instance “$VE$ of $E$” means “the variable-environment component of $E$” and “$m$ of $S$” means “the structure name of $S$”.

Moreover, when a tuple contains a finite map we shall “apply” the tuple to an argument, relying on the syntactic class of the argument to determine the relevant function. For instance $C(tycon)$ means $(TE$ of $C)tycon$.

A particular case needs mention: $C(con)$ is taken to stand for $(VE$ of $C)con$; similarly, $C(excon)$ is taken to stand for $(VE$ of $C)excon$. The type scheme of a value constructor is held in $VE$ as well as in $TE$ (where it will be recorded within a $CE$); similarly, the type of an exception constructor is held in $VE$ as well as in $EE$. Thus the re-binding of a constructor of either kind is given proper effect by accessing it in $VE$, rather than in $TE$ or in $EE$.

Finally, environments may be applied to long identifiers. For instance if $longcon = strid_1, \ldots, strid_k.con$ then $E(longcon)$ means

$$(VE$ of $(SE$ of $\ldots(SE$ of $(SE$ of $E)strid_1)strid_2\ldots)strid_k)con.$$  

**Injection:** Components may be injected into tuple classes; for example, “$VE$ in $Env$” means the environment $(\{\},\{\},VE,\{\})$.

**Modification:** The modification of one map $f$ by another map $g$, written $f+g$, has already been mentioned. It is commonly used for environment modification, for example $E + E'$. Often, empty components will be left implicit in a modification; for example $E + VE$ means $E + \{(\},\{\},VE,\{\})$. For set components, modification means union, so that $C + (T,VE)$ means

$$( (T$ of $C) \cup T, U$ of $C, (E$ of $C) + VE$$

Finally, we frequently need to modify a context $C$ by an environment $E$ (or a type environment $TE$ say), at the same time extending $T$ of $C$ to include the type names of $E$ (or of $TE$ say). We therefore define $C \oplus TE$, for example, to mean $C + (tynamesTE,TE)$.

### 4.4 Types and Type functions

A type $\tau$ is an equality type, or admits equality, if it is of one of the forms

- $\alpha$, where $\alpha$ admits equality;


\( \{ \text{lab}_1 \mapsto \tau_1, \ldots, \text{lab}_n \mapsto \tau_n \} \), where each \( \tau_i \) admits equality;

- \( \tau^{(k)} t \), where \( t \) and all members of \( \tau^{(k)} \) admit equality;
- \( (\tau') \text{ref} \).

A type function \( \theta = \Lambda \alpha^{(k)} \tau \) has arity \( k \); it must be closed – i.e. \( \text{tyvars}(\tau) \subseteq \alpha^{(k)} \) – and the bound variables must be distinct. Two type functions are considered equal if they only differ in their choice of bound variables (alpha-conversion). In particular, the equality attribute has no significance in a bound variable of a type function; for example, \( \Lambda \alpha. \alpha \to \alpha \) and \( \Lambda \beta. \beta \to \beta \) are equal type functions even if \( \alpha \) admits equality but \( \beta \) does not. Similarly, the imperative attribute has no significance in the bound variable of a type function. If \( t \) has arity \( k \), then we write \( t \) to mean \( \Lambda \alpha^{(k)} \alpha^{(k)} t \) (eta-conversion); thus \( \text{TyName} \subseteq \text{TypeFn} \). \( \theta = \Lambda \alpha^{(k)} \tau \) is an equality type function, or admits equality, if when the type variables \( \alpha^{(k)} \) are chosen to admit equality then \( \tau \) also admits equality.

We write the application of a type function \( \theta \) to a vector \( \tau^{(k)} \) of types as \( \tau^{(k)} \theta \). If \( \theta = \Lambda \alpha^{(k)} \tau \) we set \( \tau^{(k)} \theta = \tau \{ \tau^{(k)} / \alpha^{(k)} \} \) (beta-conversion).

We write \( \tau \{ \theta^{(k)}/t^{(k)} \} \) for the result of substituting type functions \( \theta^{(k)} \) for type names \( t^{(k)} \) in \( \tau \). We assume that all beta-conversions are carried out after substitution, so that for example

\[
(\tau^{(k)}t)\{\Lambda \alpha^{(k)} \tau / t\} = \tau \{ \tau^{(k)} / \alpha^{(k)} \}.
\]

A type is imperative if all type variables occurring in it are imperative.

### 4.5 Type Schemes

A type scheme \( \sigma = \forall \alpha^{(k)} \tau \) generalises a type \( \tau' \), written \( \sigma \gg \tau' \), if \( \tau' = \tau \{ \tau^{(k)} / \alpha^{(k)} \} \) for some \( \tau^{(k)} \), where each member \( \tau_i \) of \( \tau^{(k)} \) admits equality if \( \alpha_i \) does, and \( \tau_i \) is imperative if \( \alpha_i \) is imperative. If \( \sigma' = \forall \beta^{(l)} \tau' \) then \( \sigma \) generalises \( \sigma' \), written \( \sigma \gg \sigma' \), if \( \sigma \gg \tau' \) and \( \beta^{(l)} \) contains no free type variable of \( \sigma \). It can be shown that \( \sigma \gg \sigma' \) iff, for all \( \tau'' \), whenever \( \sigma' \gg \tau'' \) then also \( \sigma \gg \tau'' \).

Two type schemes \( \sigma \) and \( \sigma' \) are considered equal if they can be obtained from each other by renaming and reordering of bound type variables, and deleting type variables from the prefix which do not occur in the body. Here, in contrast to the case for type functions, the equality attribute must be preserved in renaming; for example \( \forall \alpha. \alpha \to \alpha \) and \( \forall \beta. \beta \to \beta \) are only equal if either both \( \alpha \) and \( \beta \) admit equality, or neither does. Similarly, the imperative attribute of a bound type variable of a type scheme is significant. It can be shown that \( \sigma = \sigma' \) iff \( \sigma \gg \sigma' \) and \( \sigma' \gg \sigma \).

We consider a type \( \tau \) to be a type scheme, identifying it with \( \forall () \tau \).
4.6 Scope of Explicit Type Variables

In the Core language, a type or datatype binding can explicitly introduce type variables whose scope is that binding. In the modules, a description of a value, type, or datatype may contain explicit type variables whose scope is that description. However, we still have to account for the scope of an explicit type variable occurring in the “: ty” of a typed expression or pattern or in the “of ty” of an exception binding. For the rest of this section, we consider such occurrences of type variables only.

Every occurrence of a value declaration is said to scope a set of explicit type variables determined as follows.

First, an occurrence of α in a value declaration val valbind is said to be unguarded if the occurrence is not part of a smaller value declaration within valbind. In this case we say that α occurs unguarded in the value declaration.

Then we say that α is scoped at a particular occurrence O of val valbind in a program if (1) α occurs unguarded in this value declaration, and (2) α does not occur unguarded in any larger value declaration containing the occurrence O.

Hence, associated with every occurrence of a value declaration there is a set U of the explicit type variables that are scoped at that occurrence. One may think of each occurrence of val as being implicitly decorated with such a set, for instance:

\[
\text{val}() x = (\text{let val}() \alpha \text{ Id1:}'a->'a = \text{fn } z=>z \text{ in Id1 Id1 end,} \\
\text{let val}() \alpha \text{ Id2:}'a->'a = \text{fn } z=>z \text{ in Id2 Id2 end})
\]

\[
\text{val}() a x = (\text{let val}() \alpha \text{ Id:}'a->'a = \text{fn } z=>z \text{ in Id Id end,} \\
\text{fn } z=> z: 'a)
\]

According to the inference rules in Section 4.10 the first example can be elaborated, but the second cannot since 'a is bound at the outer value declaration leaving no possibility of two different instantiations of the type of Id in the application Id Id.

4.7 Non-expansive Expressions

In order to treat polymorphic references and exceptions, the set Exp of expressions is partitioned into two classes, the expansive and the non-expansive expressions. Any variable, constructor and fn expression, possibly constrained by one or more type expressions, is non-expansive; all other expressions are said to be expansive. The idea is that the dynamic evaluation of a non-expansive expression will neither generate an exception nor extend the domain of the memory, while the evaluation of an expansive expression might.
4.8 Closure

Let \( \tau \) be a type and \( A \) a semantic object. Then \( \text{Clos}_A(\tau) \), the closure of \( \tau \) with respect to \( A \), is the type scheme \( \forall \alpha^{(k)} \cdot \tau \), where \( \alpha^{(k)} = \text{tyvars}(\tau) \setminus \text{tyvars} \cdot A \). Commonly, \( A \) will be a context \( C \). We abbreviate the total closure \( \text{Clos}(\tau) \) to \( \text{Clos}(\tau) \). If the range of a variable environment \( VE \) contains only types (rather than arbitrary type schemes) we set

\[
\text{Clos}_A(VE) = \{ \text{id} \mapsto \text{Clos}(\tau) ; \text{VE}(\text{id}) = \tau \}
\]

with a similar definition for \( \text{Clos}_A(CE) \).

Closing a variable environment \( VE \) that stems from the elaboration of a value binding \( \text{valbind} \) requires extra care to ensure type security of references and exceptions and correct scoping of explicit type variables. Recall that \( \text{valbind} \) is not allowed to bind the same variable twice. Thus, for each \( \text{var} \in \text{Dom} \cdot VE \) there is a unique \( \text{pat} = \text{exp} \) in \( \text{valbind} \) which binds \( \text{var} \). If \( \text{VE}(\text{var}) = \tau \), let \( \text{Clos}_{C,\text{valbind}}(\text{VE}(\text{var})) = \forall \alpha^{(k)} \cdot \tau \), where

\[
\alpha^{(k)} = \begin{cases} 
\text{tyvars} \tau \setminus \text{tyvars} \cdot C, & \text{if exp is non-expansive;} \\
\text{aptyvars} \tau \setminus \text{tyvars} \cdot C, & \text{if exp is expansive.}
\end{cases}
\]

Notice that the form of \( \text{valbind} \) does not affect the binding of applicative type variables, only the binding of imperative type variables.

4.9 Type Structures and Type Environments

A type structure \((\theta, CE)\) is well-formed if either \( CE = \{ \} \), or \( \theta \) is a type name \( t \). (The latter case arises, with \( CE \neq \{ \} \), in \text{datatype} declarations.) All type structures occurring in elaborations are assumed to be well-formed.

A type structure \((t, CE)\) is said to respect equality if, whenever \( t \) admits equality, then either \( t = \text{ref} \) (see Appendix C) or, for each \( CE(\text{con}) \) of the form \( \forall \alpha^{(k)}.(\tau \rightarrow \alpha^{(k)} t) \), the type function \( \Lambda \alpha^{(k)} \cdot \tau \) also admits equality. (This ensures that the equality predicate \( = \) will be applicable to a constructed value \((\text{con}, v)\) of type \( \tau^{(k)} t \) only when it is applicable to the value \( v \) itself, whose type is \( \tau^{(k)} / \alpha^{(k)} \).) A type environment \( TE \) respects equality if all its type structures do so.

Let \( TE \) be a type environment, and let \( T \) be the set of type names \( t \) such that \((t, CE)\) occurs in \( TE \) for some \( CE \neq \{ \} \). Then \( TE \) is said to maximise equality if (a) \( TE \) respects equality, and also (b) if any larger subset of \( T \) were to admit equality (without any change in the equality attribute of any type names not in \( T \)) then \( TE \) would cease to respect equality.

For any \( TE \) of the form

\[
TE = \{ \text{tycon}_i \mapsto (t_i, CE_i) ; \ 1 \leq i \leq k \},
\]
where no $CE_i$ is the empty map, and for any $E$ we define $\text{Abs}(TE, E)$ to be the environment obtained from $E$ and $TE$ as follows. First, let $\text{Abs}(TE)$ be the type environment $\{\text{tycon}_i \mapsto (t_i, \{\}) \mid 1 \leq i \leq k\}$ in which all constructor environments $CE_i$ have been replaced by the empty map. Let $t'_1, \ldots, t'_k$ be new distinct type names none of which admit equality. Then $\text{Abs}(TE, E)$ is the result of simultaneously substituting $t'_i$ for $t_i, 1 \leq i \leq k$, throughout $\text{Abs}(TE) + E$. (The effect of the latter substitution is to ensure that the use of equality on an \texttt{abstype} is restricted to the \texttt{with} part.)
4.10 Inference Rules

Each rule of the semantics allows inferences among sentences of the form

\[ A \vdash \text{phrase} \Rightarrow A' \]

where \( A \) is usually an environment or a context, \textit{phrase} is a phrase of the Core, and \( A' \) is a semantic object – usually a type or an environment. It may be pronounced “\textit{phrase} elaborates to \( A' \) in (context or environment) \( A \)”.

Some rules have extra hypotheses not of this form; they are called side conditions.

In the presentation of the rules, phrases within single angle brackets \( \langle \rangle \) are called first options, and those within double angle brackets \( \langle \langle \rangle \rangle \) are called second options. To reduce the number of rules, we have adopted the following convention:

In each instance of a rule, the first options must be either all present or all absent; similarly the second options must be either all present or all absent.

Although not assumed in our definitions, it is intended that every context \( C = T, U, E \) has the property that tynames \( E \subseteq T \). Thus \( T \) may be thought of, loosely, as containing all type names which “have been generated”. It is necessary to include \( T \) as a separate component in a context, since tynames \( E \) may not contain all the type names which have been generated; one reason is that a context \( T, \emptyset, E \) is a projection of the basis \( B = (M, T) \), \( F, G, E \) whose other components \( F \) and \( G \) could contain other such names – recorded in \( T \) but not present in \( E \).

Of course, remarks about what “has been generated” are not precise in terms of the semantic rules. But the following precise result may easily be demonstrated:

Let \( S \) be a sentence \( T, U, E \vdash \text{phrase} \Rightarrow A \) such that tynames \( E \subseteq T \), and let \( S' \) be a sentence \( T', U', E' \vdash \text{phrase}' \Rightarrow A' \) occurring in a proof of \( S \); then also tynames \( E' \subseteq T' \).

Atomic Expressions

\[
\begin{array}{c}
C \vdash \text{atexp} \Rightarrow \tau \\
\end{array}
\]

\[
\begin{align*}
C \vdash \text{scon} & \Rightarrow \text{type(scon)} \quad (1) \\
C \vdash \text{longvar} & \Rightarrow \tau \\
& \quad (2) \\
C \vdash \text{longcon} & \Rightarrow \tau \\
& \quad (3) \\
C \vdash \text{longexcon} & = \tau \\
& \quad (4)
\end{align*}
\]
\[
\begin{align*}
\langle C \vdash \text{exprow} \Rightarrow \varrho \rangle \\
C \vdash \{ \langle \text{exprow} \rangle \} \Rightarrow \{ \{ \} \} \in \text{Type} \\
C \vdash \text{dec} \Rightarrow E \quad C \oplus E \vdash \text{exp} \Rightarrow \tau \\
C \vdash \text{let} \ \text{dec in} \ \text{exp} \ \text{end} \Rightarrow \tau \\
C \vdash \text{exp} \Rightarrow \tau \\
C \vdash (\text{exp}) \Rightarrow \tau
\end{align*}
\]

Comments:

(2),(3) The instantiation of type schemes allows different occurrences of a single \textit{longvar} or \textit{longcon} to assume different types.

(6) The use of \( \oplus \), here and elsewhere, ensures that type names generated by the first sub-phrase are different from type names generated by the second sub-phrase.

Expression Rows

\[
C \vdash \text{exp} \Rightarrow \tau \quad \langle C \vdash \text{exprow} \Rightarrow \varrho \rangle \\
C \vdash \text{lab} = \text{exp} \ (, \ \text{exprow}) \Rightarrow \{ \text{lab} \mapsto \tau \} \{ \} \{ \} 
\]

Expressions

\[
C \vdash \text{atexp} \Rightarrow \tau \\
C \vdash \text{atexp} \Rightarrow \tau \\
C \vdash \text{exp} \Rightarrow \tau \quad C \vdash \text{atexp} \Rightarrow \tau' \\
C \vdash \text{exp} \text{ atexp} \Rightarrow \tau \\
C \vdash \text{exp} \Rightarrow \tau \quad C \vdash \text{ty} \Rightarrow \tau \\
C \vdash \text{exp} : \text{ty} \Rightarrow \tau \\
C \vdash \text{exp} \ \text{handle} \ \text{match} \Rightarrow \text{exn} \Rightarrow \tau \\
C \vdash \text{exp} \ \text{handle} \ \text{match} \Rightarrow \tau \\
C \vdash \text{match} \Rightarrow \tau \\
C \vdash \text{fn} \ \text{match} \Rightarrow \tau
\]

Comments:

(9) The relational symbol \( \vdash \) is overloaded for all syntactic classes (here atomic expressions and expressions).
(11) Here $\tau$ is determined by $C$ and $ty$. Notice that type variables in $ty$ cannot
be instantiated in obtaining $\tau$; thus the expression 1: ‘a will not elaborate
successfully, nor will the expression (fn $x\Rightarrow x$):’$a\rightarrow'b$. The effect of type
variables in an explicitly typed expression is to indicate exactly the degree of
polymorphism present in the expression.

(13) Note that $\tau$ does not occur in the premise; thus a raise expression has
"arbitrary" type.

**Matches**

$$
\frac{C \vdash \text{mrule} \Rightarrow \tau}{C \vdash \text{mrule} \langle | \text{match} \rangle \Rightarrow \tau}
$$

(15)

**Match Rules**

$$
\frac{C \vdash \text{pat} \Rightarrow (VE,\tau) \quad C + VE \vdash \text{exp} \Rightarrow \tau'}{C \vdash \text{pat} \Rightarrow \text{exp} \Rightarrow \tau \rightarrow \tau'}
$$

*Comment:* This rule allows new free type variables to enter the context. These
new type variables will be chosen, in effect, during the elaboration of $\text{pat}$ (i.e.,
in the inference of the first hypothesis). In particular, their choice may have to be
made to agree with type variables present in any explicit type expression occurring
within $\text{exp}$ (see rule 11).

**Declarations**

$$
\frac{C + U \vdash \text{valbind} \Rightarrow VE \quad VE' = \text{Clos}_{C,\text{valbind}}VE \quad U \triangleq \text{tyvars}VE' = \emptyset}{C \vdash \text{val}_U \text{ valbind} \Rightarrow VE' \text{ in Env}}
$$

(17)

$$
\frac{C \vdash \text{typpbind} \Rightarrow TE}{C \vdash \text{type typpbind} \Rightarrow TE \text{ in Env}}
$$

(18)

$$
\frac{C \oplus TE \vdash \text{datbind} \Rightarrow VE, TE \quad \forall(t,CE) \in \text{Ran} TE, \ t \notin (T \ of \ C) \quad TE \text{ maximises equality}}{C \vdash \text{datatype datbind} \Rightarrow (VE,TE) \text{ in Env}}
$$

(19)

$$
\frac{C \oplus TE \vdash \text{datbind} \Rightarrow VE,TE \quad \forall(t,CE) \in \text{Ran} TE, \ t \notin (T \ of \ C) \quad TE \text{ maximises equality}}{C \vdash \text{abstype datbind with dec end} \Rightarrow \text{Abs}(TE,E)}
$$

(20)

$$
\frac{C \vdash \text{exbind} \Rightarrow EE \quad VE = EE}{C \vdash \text{exception exbind} \Rightarrow (VE,EE) \text{ in Env}}
$$

(21)
\[
C \vdash dec_1 \Rightarrow E_1 \quad C \perp E_1 \vdash dec_2 \Rightarrow E_2 \\
C \vdash \text{local } dec_1 \text{ in } dec_2 \text{ end } \Rightarrow E_2 \\
C(\text{longstrid}_1) = (m_1, E_1) \quad \cdots \quad C(\text{longstrid}_n) = (m_n, E_n) \\
C \vdash \text{open } longstrid_1 \cdots longstrid_n \Rightarrow E_1 + \cdots + E_n
\]  
(22)

\[
C \vdash \{ \} \Rightarrow \{ \} \text{ in Env} \\
C \vdash dec_1 \Rightarrow E_1 \quad C \perp E_1 \vdash dec_2 \Rightarrow E_2 \\
C \vdash dec_1 \langle ; \rangle dec_2 \Rightarrow E_1 + E_2
\]  
(24)

Comments:

(17) Here \( VE \) will contain types rather than general type schemes. The closure of \( VE \) is exactly what allows variables to be used polymorphically, via rule 2. Moreover, \( U \) is the set of explicit type variables scoped at this particular occurrence of \text{val valbind}, cf. Section 4.6, page 20. The side-condition on \( U \) ensures that these explicit type variables are bound by the closure operation. On the other hand, no other explicit type variable occurring free in \( VE \) will become bound, since it must be in \( U \) of \( C \), and is therefore excluded from closure by the definition of the closure operation (Section 4.8, page 21) since \( U \) of \( C \subseteq \text{tyvars}C \).

(19),(20) The side conditions express that the elaboration of each datatype binding generates new type names and that as many of these new names as possible admit equality. Adding \( TE \) to the context on the left of the \( \vdash \) captures the recursive nature of the binding.

(20) The Abs operation was defined in Section 4.9, page 22.

(21) No closure operation is used here, since \( EE \) maps exception names to types rather than to general type schemes. Note that \( EE \) is also recorded in the \text{VarEnv} component of the resulting environment (see Section 4.3, page 18).

Value Bindings

\[
C \vdash \text{valbind} \Rightarrow VE \\
C \vdash pat \Rightarrow (VE, \tau) \\
C \vdash exp \Rightarrow \tau \\
\langle C \vdash \text{valbind} \Rightarrow VE' \rangle \\
C \vdash pat = exp \langle \text{and valbind} \rangle \Rightarrow VE \langle + VE' \rangle
\]  
(26)

\[
C + VE \vdash \text{valbind} \Rightarrow VE \\
C \vdash \text{rec valbind} \Rightarrow VE
\]  
(27)

Comments:

(26) When the option is present we have \( \text{Dom } VE \cap \text{Dom } VE' = \emptyset \) by the syntactic restrictions.
(27) Modifying $C$ by $VE$ on the left captures the recursive nature of the binding. From rule 26 we see that any type scheme occurring in $VE$ will have to be a type. Thus each use of a recursive function in its own body must be ascribed the same type.

**Type Bindings**

\[
\frac{\text{tyvarseq} = \alpha^{(k)} \quad C \vdash ty \Rightarrow \tau \quad \langle C \vdash \text{typbind} \Rightarrow TE \rangle}{C \vdash \text{tyvarseq tycon} = ty \langle \text{and typbind} \rangle \Rightarrow \{\text{tycon} \mapsto (\Lambda\alpha^{(k)} \cdot \tau, \{\})\} \langle + TE \rangle}
\]

*Comment:* The syntactic restrictions ensure that the type function $\Lambda\alpha^{(k)} \cdot \tau$ satisfies the well-formedness constraints of Section 4.4 and they ensure $\text{tycon} \notin \text{Dom} \cdot TE$.

**Data Type Bindings**

\[
\frac{\text{tyvarseq} = \alpha^{(k)} \quad C, \alpha^{(k)} t \vdash \text{conbind} \Rightarrow CE \quad \langle C \vdash \text{datbind} \Rightarrow VE, TE \rangle \quad \forall (t', CE) \in \text{Ran} \cdot TE, t \neq t'}{C \vdash \text{tyvarseq tycon} = \text{conbind} \langle \text{and datbind} \rangle \Rightarrow \text{Clo} \cdot CE (+ VE), \{\text{tycon} \mapsto (t, \text{Clo} \cdot CE)\} \langle + TE \rangle}
\]

*Comment:* The syntactic restrictions ensure $\text{Dom} \cdot VE \cap \text{Dom} \cdot CE = \emptyset$ and $\text{tycon} \notin \text{Dom} \cdot TE$.

**Constructor Bindings**

\[
\frac{\langle C \vdash ty \Rightarrow \tau' \rangle \quad \langle \langle C, \tau \vdash \text{conbind} \Rightarrow CE \rangle \rangle}{C, \tau \vdash \text{con (of ty)} \Rightarrow \langle \langle \mid \text{conbind} \rangle \rangle \Rightarrow \{\text{con} \mapsto \tau\} \langle + \{\text{con} \mapsto \tau' \mapsto \tau\} \rangle \langle + CE \rangle}
\]

*Comment:* By the syntactic restrictions $\text{con} \notin \text{Dom} \cdot CE$.

**Exception Bindings**

\[
\frac{\langle C \vdash ty \Rightarrow \tau \quad \tau \text{ is imperative} \rangle \quad \langle \langle C \vdash \text{exbind} \Rightarrow EE \rangle \rangle}{C \vdash \text{excon (of ty)} \Rightarrow \langle \langle \text{and exbind} \rangle \rangle \Rightarrow \{\text{excon} \mapsto \text{exn}\} \langle + \{\text{excon} \mapsto \tau \mapsto \text{exn}\} \rangle \langle + EE \rangle}
\]

\[
C(\text{longexcon}) = \tau \quad \langle C \vdash \text{exbind} \Rightarrow EE \rangle
\]

\[
C \vdash \text{excon} = \text{longexcon (and exbind)} \Rightarrow \{\text{excon} \mapsto \tau\} \langle + EE \rangle
\]

*Comments:*

(31) Notice that $\tau$ must not contain any applicative type variables.
(31), (32) There is a unique EE, for each C and exbind, such that $C \vdash \text{exbind} \Rightarrow EE$.

**Atomic Patterns**

$$C \vdash \text{atpat} \Rightarrow (VE, \tau)$$

(33)

$$C \vdash \_ \Rightarrow ([], \tau)$$

$$C \vdash scon \Rightarrow ([], \text{type}(scon))$$

(34)

$$C \vdash \text{var} \Rightarrow ([\text{var} \mapsto \tau], \tau)$$

(35)

$$C(\text{longcon}) \succ \tau^{(k)} t$$

$$C \vdash \text{longcon} \Rightarrow ([], \tau^{(k)} t)$$

(36)

$$C(\text{longexcon}) = \text{exn}$$

$$C \vdash \text{longexcon} \Rightarrow ([], \text{exn})$$

(37)

$$\langle C \vdash \text{patrow} \Rightarrow (VE, \varrho) \rangle$$

$$C \vdash \{ \langle \text{patrow} \rangle \} \Rightarrow (\{\}, \langle + \text{VE} \rangle, \{\} \langle + \varrho \rangle \text{ in Type } )$$

(38)

$$C \vdash \text{pat} \Rightarrow (VE, \tau)$$

$$C \vdash (\text{pat}) \Rightarrow (VE, \tau)$$

(39)

**Comments:**

(35) Note that var can assume a type, not a general type scheme.

**Pattern Rows**

$$C \vdash \text{patrow} \Rightarrow (VE, \varrho)$$

(40)

$$C \vdash \ldots \Rightarrow ([], \varrho)$$

$$C \vdash \text{pat} \Rightarrow (VE, \tau)$$

$$\langle C \vdash \text{patrow} \Rightarrow (VE', \varrho) \rangle \quad \text{lab} \notin \text{Dom} \varrho$$

$$C \vdash \text{lab} = \text{pat} (', \text{patrow}) \Rightarrow (VE(\langle + \text{VE'} \rangle, \{\text{lab} \mapsto \tau\} \langle + \varrho \rangle))$$

(41)

**Comment:**

(41) By the syntactic restrictions, DomVE \cap DomVE' = \emptyset.
Patterns

\[
C \vdash \text{atpat} \Rightarrow (VE, \tau) \\
C \vdash \text{atpat} \Rightarrow (VE, \tau')
\]

\[
\frac{C(\text{longcon}) \vdash \tau' \rightarrow \tau \quad C \vdash \text{atpat} \Rightarrow (VE, \tau')}{C \vdash \text{longcon atpat} \Rightarrow (VE, \tau)}
\]

\[
\frac{C(\text{longexcon}) = \tau \rightarrow \text{exn} \quad C \vdash \text{atpat} \Rightarrow (VE, \tau)}{C \vdash \text{longexcon atpat} \Rightarrow (VE, \text{exn})}
\]

\[
\frac{C \vdash \text{pat} \Rightarrow (VE, \tau) \quad C \vdash \text{ty} \Rightarrow \tau}{C \vdash \text{pat : ty} \Rightarrow (VE, \tau)}
\]

\[
\frac{C \vdash \text{var} \Rightarrow (VE, \tau) \quad \langle C \vdash \text{ty} \Rightarrow \tau \rangle}{C \vdash \text{var}\langle : \text{ty} \rangle \text{ as pat} \Rightarrow (VE + VE', \tau)}
\]

Comments:

(46) By the syntactic restrictions, \(\text{Dom}VE \cap \text{Dom}VE' = \emptyset\).

Type Expressions

\[
\frac{\text{tyvar} = \alpha}{C \vdash \text{tyvar} \Rightarrow \alpha}
\]

\[
\frac{\langle C \vdash \text{tyrow} \Rightarrow \theta \rangle}{C \vdash \{ \langle \text{tyrow} \rangle \} \Rightarrow \{ \langle \text{tyrow} \rangle \} \text{ in Type}}
\]

\[
\text{tyseq} = \text{ty}_1 \cdots \text{ty}_k \\
C \vdash \text{ty}_i \Rightarrow \tau_i (1 \leq i \leq k)
\]

\[
\frac{C(\text{longtycon}) = (\theta, CE)}{C \vdash \text{tyseq longtycon} \Rightarrow \tau^{(k)}\theta}
\]

\[
\frac{C \vdash \text{ty} \Rightarrow \tau \quad C \vdash \text{ty}' \Rightarrow \tau'}{C \vdash \text{ty} \rightarrow \text{ty}' \Rightarrow \tau \rightarrow \tau'}
\]

\[
\frac{C \vdash \text{ty} \Rightarrow \tau}{C \vdash (\text{ty}) \Rightarrow \tau}
\]

Comments:

(49) Recall that for \(\tau^{(k)}\theta\) to be defined, \(\theta\) must have arity \(k\).
Type-expression Rows

\[
\begin{align*}
C \vdash ty \Rightarrow \tau & \quad \langle C \vdash tyrow \Rightarrow \phi \rangle \\
C \vdash \text{lab} : ty \langle \cdot , tyrow \rangle & \Rightarrow \{ \text{lab} \rightarrow \tau \}(+ \phi)
\end{align*}
\]

(52)

Comment: The syntactic constraints ensure \( \text{lab} \notin \text{Dom} \phi \).

4.11 Further Restrictions

There are a few restrictions on programs which should be enforced by a compiler, but are better expressed apart from the preceding Inference Rules. They are:

1. For each occurrence of a record pattern containing a record wildcard, i.e. of the form \( \{ \text{lab}_1=\text{pat}_1, \ldots, \text{lab}_m=\text{pat}_m, \ldots \} \) the program context must determine uniquely the domain \( \{ \text{lab}_1, \ldots, \text{lab}_n \} \) of its record type, where \( m \leq n \); thus, the context must determine the labels \( \{ \text{lab}_{m+1}, \ldots, \text{lab}_n \} \) of the fields to be matched by the wildcard. For this purpose, an explicit type constraint may be needed. This restriction is necessary to ensure the existence of principal type schemes.

2. In a match of the form \( \text{pat}_1 \Rightarrow \text{exp}_1 | \cdots | \text{pat}_n \Rightarrow \text{exp}_n \) the pattern sequence \( \text{pat}_1, \ldots, \text{pat}_n \) should be irredundant; that is, each \( \text{pat}_i \) must match some value (of the right type) which is not matched by \( \text{pat}_i \) for any \( i < j \). In the context \( \text{fn match} \), the match must also be exhaustive; that is, every value (of the right type) must be matched by some \( \text{pat}_i \). The compiler must give warning on violation of these restrictions, but should still compile the match. The restrictions are inherited by derived forms; in particular, this means that in the function binding \( \text{var atpat}_1 \cdots \text{atpat}_n ; ty \) = \( \text{exp} \) (consisting of one clause only), each separate \( \text{atpat}_i \) should be exhaustive by itself.

4.12 Principal Environments

The notion of enrichment, \( E \succ E' \), between environments \( E = (SE, TE, VE, EE) \) and \( E' = (SE', TE', VE', EE') \) is defined in Section 5.11. For the present section, \( E \succ E' \) may be taken to mean \( SE = SE' = \{ \}, TE = TE', EE = EE', \text{Dom}VE = \text{Dom}VE' \) and, for each \( id \in \text{Dom}VE, VE(id) \succ VE'(id) \).

Let \( C \) be a context, and suppose that \( C \vdash \text{dec} \Rightarrow E \) according to the preceding Inference Rules. Then \( E \) is principal (for \( \text{dec} \) in the context \( C \)) if, for all \( E' \) for which \( C \vdash \text{dec} \Rightarrow E' \), we have \( E \succ E' \). We claim that if \( \text{dec} \) elaborates to any environment in \( C \) then it elaborates to a principal environment in \( C \). Strictly, we must allow for the possibility that type names and imperative type variables which do not occur in \( C \) are chosen differently for \( E \) and \( E' \). The stated claim is therefore made up to such variation.
5 Static Semantics for Modules

5.1 Semantic Objects

The simple objects for Modules static semantics are exactly as for the Core. The
compound objects are those for the Core, augmented by those in Figure 11.

\[
\begin{align*}
M & \in \text{StrNameSet} = \text{Fin}(\text{StrName}) \\
N \text{ or } (M, T) & \in \text{NameSet} = \text{StrNameSet} \times \text{TyNameSet} \\
\Sigma \text{ or } (N)S & \in \text{Sig} = \text{NameSet} \times \text{Str} \\
\Phi \text{ or } (N)(S, (N)S') & \in \text{FunSig} = \text{NameSet} \times (\text{Str} \times \text{Sig}) \\
G & \in \text{SigEnv} = \text{SigId} \xrightarrow{\text{fin}} \text{Sig} \\
F & \in \text{FunEnv} = \text{FunId} \xrightarrow{\text{fin}} \text{FunSig} \\
B \text{ or } N, F, G, E & \in \text{Basis} = \text{NameSet} \times \text{FunEnv} \times \text{SigEnv} \times \text{Env}
\end{align*}
\]

Figure 11: Further Compound Semantic Objects

The prefix \(N\), in signatures and functor signatures, binds both type names and structure names. We shall always consider a set \(N\) of names as partitioned into a pair \((M, T)\) of sets of the two kinds of name.

It is sometimes convenient to work with an arbitrary semantic object \(A\), or assembly \(A\) of such objects. As with the function tnames, \(\text{strnames}(A)\) and \(\text{names}(A)\) denote respectively the set of structure names and the set of names occurring free in \(A\).

Certain operations require a change of bound names in semantic objects; see for example Section 5.7. When bound type names are changed, we demand that all of their attributes (i.e. imperative, equality and arity) are preserved.

For any structure \(S = (m, (SE, TE, VE, EE))\) we call \(m\) the structure name or name of \(S\); also, the proper substructures of \(S\) are the members of \(\text{Ran} SE\) and their proper substructures. The substructures of \(S\) are \(S\) itself and its proper substructures. The structures occurring in an object or assembly \(A\) are the structures and substructures from which it is built.

The operations of projection, injection and modification are as for the Core. Moreover, we define \(C\) of \(B\) to be the context \((T \text{ of } B, \emptyset, E \text{ of } B)\), i.e. with an empty set of explicit type variables. Also, we frequently need to modify a basis \(B\) by an environment \(E\) (or a structure environment \(SE\) say), at the same time extending \(N\) of \(B\) to include the type names and structure names of \(E\) (or of \(SE\) say). We therefore define \(B \oplus SE\), for example, to mean \(B + (\text{names } SE, SE)\).
5.2 Consistency

A set of type structures is said to be consistent if, for all \((\theta_1, CE_1)\) and \((\theta_2, CE_2)\) in the set, if \(\theta_1 = \theta_2\) then

\[
CE_1 = \{\} \text{ or } CE_2 = \{\} \text{ or } \text{Dom} CE_1 = \text{Dom} CE_2
\]

A semantic object \(A\) or assembly \(A\) of objects is said to be consistent if (after changing bound names to make all nameset prefixes in \(A\) disjoint) for all \(S_1\) and \(S_2\) occurring in \(A\) and for every \(\text{longstrid}\) and every \(\text{longtycon}\)

1. If \(m\) of \(S_1 = m\) of \(S_2\), and both \(S_1(\text{longstrid})\) and \(S_2(\text{longstrid})\) exist, then

\[
m \text{ of } S_1(\text{longstrid}) = m \text{ of } S_2(\text{longstrid})
\]

2. If \(m\) of \(S_1 = m\) of \(S_2\), and both \(S_1(\text{longtycon})\) and \(S_2(\text{longtycon})\) exist, then

\[
\theta \text{ of } S_1(\text{longtycon}) = \theta \text{ of } S_2(\text{longtycon})
\]

3. The set of all type structures in \(A\) is consistent

As an example, a functor signature \((N)(S, (N')S')\) is consistent if, assuming first that \(N \cap N' = \emptyset\), the assembly \(A = \{S, S'\}\) is consistent.

We may loosely say that two structures \(S_1\) and \(S_2\) are consistent if \(\{S_1, S_2\}\) is consistent, but must remember that this is stronger than the assertion that \(S_1\) is consistent and \(S_2\) is consistent.

Note that if \(A\) is a consistent assembly and \(A' \subset A\) then \(A'\) is also a consistent assembly.

5.3 Well-formedness

A signature \((N)S\) is well-formed if \(N \subseteq \text{names} S\), and also, whenever \((m, E)\) is a substructure of \(S\) and \(m \notin N\), then \(N \cap (\text{names} E) = \emptyset\). A functor signature \((N)(S, (N')S')\) is well-formed if \((N)S\) and \((N')S'\) are well-formed, and also, whenever \((m', E')\) is a substructure of \(S'\) and \(m' \notin N \cup N'\), then \((N \cup N') \cap (\text{names} E') = \emptyset\).

An object or assembly \(A\) is well-formed if every type environment, signature and functor signature occurring in \(A\) is well-formed.

5.4 Cycle-freedom

An object or assembly \(A\) is cycle-free if it contains no cycle of structure names; that is, there is no sequence

\[
m_0, \ldots, m_{k-1}, m_k = m_0 \quad (k > 0)
\]

of structure names such that, for each \(i\) (\(0 \leq i < k\)) some structure with name \(m_i\) occurring in \(A\) has a proper substructure with name \(m_{i+1}\).
5.5 Admissibility

An object or assembly \( A \) is \textit{admissible} if it is consistent, well-formed and cycle-free. Henceforth it is assumed that all objects mentioned are admissible. We also require that

1. In every sentence \( A \vdash \text{phrase} \Rightarrow A' \) inferred by the rules given in Section 5.14, the assembly \( \{A, A'\} \) is admissible.

2. In the special case of a sentence \( B \vdash \text{sigexp} \Rightarrow S \), we further require that the assembly consisting of all semantic objects occurring in the entire inference of this sentence be admissible. This is important for the definition of principal signatures in Section 5.13.

In our semantic definition we have not undertaken to indicate how admissibility should be checked in an implementation.

5.6 Type Realisation

A \textit{type realisation} is a map \( \varphi_{\text{Ty}} : \text{TyName} \rightarrow \text{TypeFcn} \) such that \( t \) and \( \varphi_{\text{Ty}}(t) \) have the same arity, and if \( t \) admits equality then so does \( \varphi_{\text{Ty}}(t) \).

The \textit{support} \( \text{Supp} \varphi_{\text{Ty}} \) of a type realisation \( \varphi_{\text{Ty}} \) is the set of type names \( t \) for which \( \varphi_{\text{Ty}}(t) \neq t \).

5.7 Realisation

A \textit{realisation} is a function \( \varphi \) of names, partitioned into a type realisation \( \varphi_{\text{Ty}} : \text{TyName} \rightarrow \text{TypeFcn} \) and a function \( \varphi_{\text{Str}} : \text{StrName} \rightarrow \text{StrName} \). The \textit{support} \( \text{Supp} \varphi \) of a realisation \( \varphi \) is the set of names \( n \) for which \( \varphi(n) \neq n \). The \textit{yield} \( \text{Yield} \varphi \) of a realisation \( \varphi \) is the set of names which occur in some \( \varphi(n) \) for which \( n \in \text{Supp} \varphi \).

Realisations \( \varphi \) are extended to apply to all semantic objects; their effect is to replace each name \( n \) by \( \varphi(n) \). In applying \( \varphi \) to an object with bound names, such as a signature \( (N)S \), first bound names must be changed so that, for each binding prefix \( (N) \),

\[
N \cap (\text{Supp} \varphi \cup \text{Yield} \varphi) = \emptyset .
\]

5.8 Type Explication

A signature \( (N)S \) is \textit{type-explicit} if, whenever \( t \in N \) and occurs free in \( S \), then some substructure of \( S \) contains a type environment \( TE \) such that \( TE(\text{tycon}) = (t, CE) \) for some \text{tycon} and some \text{CE}. 

5.9 Signature Instantiation

A structure \( S_2 \) is an instance of a signature \( \Sigma_1 = (N_1)S_1 \), written \( \Sigma_1 \geq S_2 \), if there exists a realisation \( \varphi \) such that \( \varphi(S_1) = S_2 \) and \( \text{Supp } \varphi \subseteq N_1 \). (Note that if \( \Sigma_1 \) is type-explicit then there is at most one such \( \varphi \).) A signature \( \Sigma_2 = (N_2)S_2 \) is an instance of \( \Sigma_1 = (N_1)S_1 \), written \( \Sigma_1 \geq \Sigma_2 \), if \( \Sigma_1 \geq S_2 \) and \( N_2 \cap \text{names}(\Sigma_1) = \emptyset \). It can be shown that \( \Sigma_1 \geq \Sigma_2 \) iff, for all \( S \), whenever \( \Sigma_2 \geq S \) then \( \Sigma_1 \geq S \).

5.10 Functor Signature Instantiation

A pair \( (S_1, (N^1)S') \) is called a functor instance. Given \( \Phi = (N_1)(S_1, (N^1_1)S') \), a functor instance \( (S_2, (N^2_2)S_2') \) is an instance of \( \Phi \), written \( \Phi \geq (S_2, (N^2_2)S_2') \), if there exists a realisation \( \varphi \) such that \( \varphi(S_1, (N^1_1)S') = (S_2, (N^2_2)S_2') \) and \( \text{Supp } \varphi \subseteq N_1 \).

5.11 Enrichment

In matching a structure to a signature, the structure will be allowed both to have more components, and to be more polymorphic, than (an instance of) the signature. Precisely, we define enrichment of structures, environments and type structures by mutual recursion as follows.

A structure \( S_1 = (m_1, E_1) \) enriches another structure \( S_2 = (m_2, E_2) \), written \( S_1 \triangleright S_2 \), if

1. \( m_1 = m_2 \)
2. \( E_1 \triangleright E_2 \)

An environment \( E_1 = (SE_1, TE_1, VE_1, EE_1) \) enriches another environment \( E_2 = (SE_2, TE_2, VE_2, EE_2) \), written \( E_1 \triangleright E_2 \), if

1. \( \text{Dom } SE_1 \supseteq \text{Dom } SE_2 \), and \( SE_1(strid) \triangleright SE_2(strid) \) for all \( strid \in \text{Dom } SE_2 \)
2. \( \text{Dom } TE_1 \supseteq \text{Dom } TE_2 \), and \( TE_1(\text{tycon}) \triangleright TE_2(\text{tycon}) \) for all \( \text{tycon} \in \text{Dom } TE_2 \)
3. \( \text{Dom } VE_1 \supseteq \text{Dom } VE_2 \), and \( VE_1(id) \triangleright VE_2(id) \) for all \( id \in \text{Dom } VE_2 \)
4. \( \text{Dom } EE_1 \supseteq \text{Dom } EE_2 \), and \( EE_1(\text{excon}) = EE_2(\text{excon}) \) for all \( \text{excon} \in \text{Dom } EE_2 \)

Finally, a type structure \((\theta_1, CE_1)\) enriches another type structure \((\theta_2, CE_2)\), written \((\theta_1, CE_1) \triangleright (\theta_2, CE_2)\), if

1. \( \theta_1 = \theta_2 \)
2. Either \( CE_1 = CE_2 \) or \( CE_2 = {} \)
5.12 Signature Matching

A structure $S$ matches a signature $\Sigma_1$ if there exists a structure $S^-$ such that $\Sigma_1 \geq S^- \prec S$. Thus matching is a combination of instantiation and enrichment. There is at most one such $S^-$, given $\Sigma_1$ and $S$. Moreover, writing $\Sigma_1 = (N_1)S_1$, if $\Sigma_1 \geq S^-$ then there exists a realisation $\varphi$ with $\text{Supp } \varphi \subseteq N_1$ and $\varphi(S_1) = S^-$. We shall then say that $S$ matches $\Sigma_1$ via $\varphi$. (Note that if $\Sigma_1$ is type-explicit then $\varphi$ is uniquely determined by $\Sigma_1$ and $S$.)

A signature $\Sigma_2$ matches a signature $\Sigma_1$ if for all structures $S$, if $S$ matches $\Sigma_2$ then $S$ matches $\Sigma_1$. It can be shown that $\Sigma_2 = (N_2)S_2$ matches $\Sigma_1 = (N_1)S_1$ if and only if there exists a realisation $\varphi$ with $\text{Supp } \varphi \subseteq N_1$ and $\varphi(S_1) \prec S_2$ and $N_2 \cap \text{names } \Sigma_1 = \emptyset$.

5.13 Principal Signatures

The definitions in this section concern the elaboration of signature expressions; more precisely they concern inferences of sentences of the form $B \vdash \text{sigexp} \Rightarrow S$, where $S$ is a structure and $B$ is a basis. Recall, from Section 5.5, that the assembly of all semantic objects in such an inference must be admissible.

For any basis $B$ and any structure $S$, we say that $B$ covers $S$ if for every substructure $(m, E)$ of $S$ such that $m \in N$ of $B$:

1. For every structure identifier $\text{strid} \in \text{Dom } E$, $B$ contains a substructure $(m, E')$ with $m$ free and $\text{strid} \in \text{Dom } E'$

2. For every type constructor $\text{tycon} \in \text{Dom } E$, $B$ contains a substructure $(m, E')$ with $m$ free and $\text{tycon} \in \text{Dom } E'$

(This condition is not a consequence of consistency of $\{B, S\}$; informally, it states that if $S$ shares a substructure with $B$, then $S$ mentions no more components of the substructure than $B$ does.)

We say that a signature $(N)S$ is principal for $\text{sigexp}$ in $B$ if, choosing $N$ so that $(N \text{ of } B) \cap N = \emptyset$,

1. $B$ covers $S$

2. $B \vdash \text{sigexp} \Rightarrow S$

3. Whenever $B \vdash \text{sigexp} \Rightarrow S'$, then $(N)S \geq S'$

We claim that if $\text{sigexp}$ elaborates in $B$ to some structure covered by $B$, then it possesses a principal signature in $B$.

Analogous to the definition given for type environments in Section 4.9, we say that a semantic object $A$ respects equality if every type environment occurring in $A$ respects equality.
Now let us assume that $\textit{sigexp}$ possesses a principal signature $\Sigma_0 = (N_0)S_0$ in $B$. We wish to define, in terms of $\Sigma_0$, another signature $\Sigma$ which provides more information about the equality attributes of structures which will match $\Sigma_0$. To this end, let $T_0$ be the set of type names $t \in N_0$ which do not admit equality, and such that $(t, CE)$ occurs in $S_0$ for some $CE \neq \{}$. Then we say $\Sigma$ is \textit{equality-principal} for $\textit{sigexp}$ in $B$ if

1. $\Sigma$ respects equality

2. $\Sigma$ is obtained from $\Sigma_0$ just by making as many members of $T_0$ admit equality as possible, subject to 1. above

It is easy to show that, if any such $\Sigma$ exists, it is determined uniquely by $\Sigma_0$; moreover, $\Sigma$ exists if $\Sigma_0$ itself respects equality.
5.14 Inference Rules

As for the Core, the rules of the Modules static semantics allow sentences of the form

\[ A \vdash \text{phrase} \Rightarrow A' \]

to be inferred, where in this case \( A \) is either a basis, a context or an environment and \( A' \) is a semantic object. The convention for options is as in the Core semantics.

Although not assumed in our definitions, it is intended that every basis \( B = N, F, G, E \) in which a topdec is elaborated has the property that names \( F \cup \text{names} \cup G \cup \text{names} E \subseteq N \). This is not the case for bases in which signature expressions and specifications are elaborated, but the following Theorem can be proved:

Let \( S \) be an inferred sentence \( B \vdash \text{topdec} \Rightarrow B' \) in which \( B \) satisfies the above condition. Then \( B' \) also satisfies the condition.

Moreover, if \( S' \) is a sentence of the form \( B'' \vdash \text{phrase} \Rightarrow A \) occurring in a proof of \( S \), where \text{phrase} is either a structure expression or a structure-level declaration, then \( B'' \) also satisfies the condition.

Finally, if \( T, U, E \vdash \text{phrase} \Rightarrow A \) occurs in a proof of \( S \), where \text{phrase} is a phrase of the Core, then tnames \( E \subseteq T \).

Structure Expressions

\[ B \vdash \text{strexp} \Rightarrow S \]

\[ B \vdash \text{strdec} \Rightarrow E \quad m \notin (N \text{ of } B) \cup \text{names } E \]

\[ B \vdash \text{struct strdec end} \Rightarrow (m, E) \] (53)

\[ B(\text{longstrid}) = S \]

\[ B \vdash \text{longstrid} \Rightarrow S \] (54)

\[ B \vdash \text{strexp} \Rightarrow S \]

\[ B(\text{funid}) \geq (S'' \text{ of } (N'' \text{ of } S)') \quad S \sim S'' \]

\[ (N \text{ of } B) \cap N'' = \emptyset \]

\[ B \vdash \text{funid (strexp)} \Rightarrow S' \] (55)

\[ B \vdash \text{strdec} \Rightarrow E \quad B \oplus E \vdash \text{strexp} \Rightarrow S \]

\[ B \vdash \text{let strdec in strexp end} \Rightarrow S \] (56)

Comments:

(53) The side condition ensures that each generative structure expression receives a new name. If the expression occurs in a functor body the structure name will be bound by \((N')\) in rule 99; this will ensure that for each application of the functor, by rule 55, a new distinct name will be chosen for the structure generated.
(55) The side condition \( (N \cap N') \cap \emptyset = \emptyset \) can always be satisfied by renaming bound names in \( (N')S' \) thus ensuring that the generated structures receive new names.

Let \( B(funid) = (N)(S_f, (N')S'_f) \). Assuming that \( (N)S_f \) is type-explicit, the realisation \( \varphi \) for which \( \varphi(S_f, (N')S'_f) = (S'', (N')S') \) is uniquely determined by \( S \), since \( S \rhd S'' \) can only hold if the type names and structure names in \( S \) and \( S'' \) agree. Recall that enrichment \( \rhd \) allows more components and more polymorphism, while instantiation \( \geq \) does not.

Sharing between argument and result specified in the declaration of the functor \( funid \) is represented by the occurrence of the same name in both \( S_f \) and \( S'_f \), and this repeated occurrence is preserved by \( \varphi \), yielding sharing between the argument structure \( S \) and the result structure \( S' \) of this functor application.

(56) The use of \( \oplus \), here and elsewhere, ensures that structure and type names generated by the first sub-phrase are distinct from names generated by the second sub-phrase.

\[ B \vdash strdec \Rightarrow E \]

**Structure-level Declarations**

\[ C \text{ of } B \vdash dec \Rightarrow E \quad E \text{ principal for } dec \text{ in } (C \text{ of } B) \]

\[ B \vdash dec \Rightarrow E \]  \hspace{1cm} (57)  

\[ B \vdash strbind \Rightarrow SE \]

\[ B \vdash \text{structure } strbind \Rightarrow SE \text{ in Env} \]

\[ B \vdash strdec_1 \Rightarrow E_1 \quad B \oplus E_1 \vdash strdec_2 \Rightarrow E_2 \]

\[ B \vdash \text{local } strdec_1 \text{ in } strdec_2 \text{ end} \Rightarrow E_2 \]  \hspace{1cm} (59)  

\[ B \vdash \Rightarrow \{ \} \text{ in Env} \]  \hspace{1cm} (60)  

\[ B \vdash strdec_1 \Rightarrow E_1 \quad B \oplus E_1 \vdash strdec_2 \Rightarrow E_2 \]

\[ B \vdash \text{strdec}_1 (; \text{ strdec}_2) \Rightarrow E_1 + E_2 \]  \hspace{1cm} (61)  

**Comments:**

(57) The side condition ensures that all type schemes in \( E \) are as general as possible.
Structure Bindings

\[
B \vdash strbind \Rightarrow SE
\]

\[
B \vdash strexp \Rightarrow S \quad \langle B \vdash sigexp \Rightarrow \Sigma, \, \Sigma \geq S' \prec S \rangle
\]

\[
\langle\langle B+\text{names} \vdash strbind \Rightarrow SE \rangle\rangle
\]

\[
B \vdash \text{strid} \; \langle : \; \text{sigexp} \rangle = \text{strexp} \; \langle \langle \text{and} \; \text{strbind} \rangle \rangle \Rightarrow \{ \text{strid} \mapsto S(') \} \; \langle\{+ \; SE\} \rangle
\]  

(62)

**Comment:** If present, \( \text{sigexp} \) has the effect of restricting the view which \( \text{strid} \) provides of \( S \) while retaining sharing of names. The notation \( S(') \) means \( S' \), if the first option is present, and \( S \) if not.

Signature Expressions

\[
B \vdash \text{spec} \Rightarrow E
\]

\[
B \vdash \text{sig spec end} \Rightarrow (m, E)
\]  

(63)

\[
B(\text{sigid}) \geq S
\]

\[
B \vdash \text{sigid} \Rightarrow S
\]  

(64)

**Comments:**

(63) In contrast to rule 53, \( m \) is not here required to be new. The name \( m \) may be chosen to achieve the sharing required in rule 88, or to achieve the enrichment side conditions of rule 62 or 99. The choice of \( m \) must result in an admissible object.

(64) The instance \( S \) of \( B(\text{sigid}) \) is not determined by this rule, but – as in rule 63 – the instance may be chosen to achieve sharing properties or enrichment conditions.

\[
B \vdash \text{sigexp} \Rightarrow \Sigma
\]

\[
B \vdash \text{sigexp} \Rightarrow S \quad (N)S \text{ equality-principal for sigexp in } B
\]

\[
(N)S \text{ type-explicit}
\]

\[
B \vdash \text{sigexp} \Rightarrow (N)S
\]  

(65)

**Comment:** A signature expression \( \text{sigexp} \) which is an immediate constituent of a structure binding, a signature binding, a functor binding or a functor signature is elaborated to an equality-principal and type-explicit signature, see rules 62, 69, 95 and 99. By contrast, signature expressions occurring in structure descriptions are elaborated to structures using the liberal rules 63 and 64, see rule 87, so that names can be chosen to achieve sharing, when necessary.
Signature Declarations

\[ B \vdash \text{sig} \Rightarrow G \]

(66) The first closure restriction of Section 3.6 can be enforced by replacing the
\( B \) in the premise by \( B_0 + G \) of \( B \).

(68) A signature declaration does not create any new structures or types; hence
the use of \( + \) instead of \( \oplus \).

Signature Bindings

\[ B \vdash \text{sig} \Rightarrow G \]

Comment: The condition that \( \Sigma \) be equality-principal, implicit in the first premise,
ensures that the signature found is as general as possible given the sharing con-
straints present in \( \text{sig} \).

Specifications

\[ B \vdash \text{spec} \Rightarrow E \]

\[ C \text{ of } B \vdash \text{val} \Rightarrow \text{VE} \]

\[ B \vdash \text{val} \Rightarrow \text{val} \Rightarrow \text{Clos} \text{VE} \text{ in Env} \]

(70)

\[ C \text{ of } B \vdash \text{ty} \Rightarrow \text{TE} \]

\[ B \vdash \text{ty} \Rightarrow \text{ty} \Rightarrow \text{TE} \text{ in Env} \]

(71)

\[ C \text{ of } B \vdash \text{ty} \Rightarrow \text{TE} \quad \forall (\theta, \text{CE}) \in \text{Ran} \text{TE}, \theta \text{ admits equality} \]

\[ B \vdash \text{eq} \Rightarrow \text{ty} \Rightarrow \text{TE} \text{ in Env} \]

(72)

\[ C \text{ of } B + \text{TE} \vdash \text{dat} \Rightarrow \text{VE}, \text{TE} \]

\[ B \vdash \text{dat} \Rightarrow \text{dat} \Rightarrow \text{VE}, \text{TE} \text{ in Env} \]

(73)

\[ C \text{ of } B \vdash \text{ex} \Rightarrow \text{EE} \quad \text{VE} = \text{EE} \]

\[ B \vdash \text{ex} \Rightarrow \text{ex} \Rightarrow \text{VE}, \text{EE} \text{ in Env} \]

(74)

\[ B \vdash \text{st} \Rightarrow \text{SE} \]

\[ B \vdash \text{str} \Rightarrow \text{se} \text{ in Env} \]

(75)
5.14 Inference Rules

\[
\begin{align*}
B \vdash \text{shareq} & \Rightarrow \{\} & (76) \\
B \vdash \text{sharing} \ \text{shareq} & \Rightarrow \{\} \text{ in Env} \\
\end{align*}
\]

\[
\begin{align*}
B \vdash \text{spec}_1 & \Rightarrow E_1 & B + E_1 \vdash \text{spec}_2 & \Rightarrow E_2 \\
\text{local } \text{spec}_1 \text{ in spec}_2 \text{ end} & \Rightarrow E_2 \\
\end{align*}
\]

\[
\begin{align*}
B(\text{longstrid}_1) = (m_1, E_1) & \cdots \ B(\text{longstrid}_n) = (m_n, E_n) \\
B \vdash \text{open} \ \text{longstrid}_1 \cdots \text{longstrid}_n & \Rightarrow E_1 + \cdots + E_n \\
\end{align*}
\]

\[
\begin{align*}
B(\text{sigid}_1) \geq (m_1, E_1) & \cdots \ B(\text{sigid}_n) \geq (m_n, E_n) \\
B \vdash \text{include} \ \text{sigid}_1 \cdots \text{sigid}_n & \Rightarrow E_1 + \cdots + E_n \\
\end{align*}
\]

\[
\begin{align*}
\text{Env} & \Rightarrow \{\} \\
B \vdash \text{spec}_1 & \Rightarrow E_1 & B + E_1 \vdash \text{spec}_2 & \Rightarrow E_2 \\
\text{spec}_1 \langle ; \rangle \text{ spec}_2 & \Rightarrow E_1 + E_2 \\
\end{align*}
\]

Comments:

(70) \(VE\) is determined by \(B\) and \(valdesc\).

(71)–(73) The type functions in \(TE\) may be chosen to achieve the sharing hypothesis of rule 89 or the enrichment conditions of rules 62 and 99. In particular, the type names in \(TE\) in rule 73 need not be new. Also, in rule 71 the type functions in \(TE\) may admit equality.

(74) \(EE\) is determined by \(B\) and \(exdesc\) and contains monotypes only.

(79) The names \(m_i\) in the instances may be chosen to achieve sharing or enrichment conditions.

Value Descriptions

\[
\begin{align*}
C \vdash \text{valdesc} & \Rightarrow VE \\
C \vdash \text{ty} & \Rightarrow \tau & \langle C \vdash \text{valdesc} \Rightarrow VE \rangle \\
C \vdash \text{var} : \text{ty} \langle \text{and valdesc} \rangle & \Rightarrow \{\text{var} \mapsto \tau\} \langle + VE \rangle \\
\end{align*}
\]

Type Descriptions

\[
\begin{align*}
C \vdash \text{tydesc} & \Rightarrow TE \\
\text{tyvarseq} = \alpha^{(k)} & \langle C \vdash \text{tydesc} \Rightarrow TE \rangle \quad \text{arity } \theta = k \\
C \vdash \text{tyvarseq tycon} \langle \text{and tydesc} \rangle & \Rightarrow \{\text{tycon} \mapsto (\theta, \{\})\} \langle + TE \rangle \\
\end{align*}
\]

Comment: Note that any \(\theta\) of arity \(k\) may be chosen but that the constructor environment in the resulting type structure must be empty. For example, \texttt{datatype s=c type t sharing s=t} is a legal specification, but the type structure bound to \(t\) does not bind any value constructors.
Datatype Descriptions
\[ C \vdash \text{datdesc} \Rightarrow VE, TE \]
\[
\begin{align*}
tyvarseq = \alpha^{(k)} & \quad C, \alpha^{(k)}t \vdash \text{condesc} \Rightarrow CE & \quad \{ C \vdash \text{datdesc} \Rightarrow VE, TE \} \quad (84) \\
C \vdash \tyvarseq \tycon = \text{condesc} \land \text{datdesc} & \Rightarrow ClosCE(+ \ VE), \{ \tycon \mapsto (t, ClosCE) \} (+ \ TE)
\end{align*}
\]

Constructor Descriptions
\[ C, \tau \vdash \text{condesc} \Rightarrow CE \]
\[
\begin{align*}
\{ C \vdash \ty \Rightarrow \tau \} & \quad \langle \{ C, \tau \vdash \text{condesc} \Rightarrow CE \} \rangle \\
C, \tau \vdash \text{con} \langle \text{of} \ ty \rangle \langle \{ \mid \text{condesc} \} \rangle & \Rightarrow \\
\{ \text{con} \mapsto \text{exn} \} (+ \{ \text{con} \mapsto \tau \Rightarrow \text{exn} \}) & \langle (+ CE) \rangle
\end{align*}
\]

Exception Descriptions
\[ C \vdash \text{exdesc} \Rightarrow EE \]
\[
\begin{align*}
C \vdash \tyvarseq \langle \{ \mid \text{excon} \} \rangle & \Rightarrow \\
C \vdash \text{excon} \langle \text{of} \ ty \rangle \langle \{ \mid \text{excon} \} \rangle & \Rightarrow \\
\{ \text{excon} \mapsto \text{exn} \} (+ \{ \text{excon} \mapsto \tau \Rightarrow \text{exn} \}) & \langle (+ EE) \rangle
\end{align*}
\]

Structure Descriptions
\[ B \vdash \text{strdesc} \Rightarrow SE \]
\[
\begin{align*}
B \vdash \text{sigexp} \Rightarrow S & \quad \langle B \vdash \text{strdesc} \Rightarrow SE \rangle \\
B \vdash \text{strid} : \text{sigexp} \land \text{strdesc} & \Rightarrow \{ \text{strid} \mapsto S \} (+ SE) \quad (87)
\end{align*}
\]

Sharing Equations
\[ B \vdash \text{shareq} \Rightarrow \{ \} \]
\[
\begin{align*}
m \text{ of } B(\text{longstrid}_1) = \cdots = m \text{ of } B(\text{longstrid}_n) & \Rightarrow \{ \} \\
B \vdash \text{longstrid}_1 = \cdots = \text{longstrid}_n & \Rightarrow \{ \} \quad (88)
\end{align*}
\]
\[
\begin{align*}
\theta \text{ of } B(\text{longtycon}_1) = \cdots = \theta \text{ of } B(\text{longtycon}_n) & \Rightarrow \{ \} \\
B \vdash \text{type} \text{longtycon}_1 = \cdots = \text{longtycon}_n & \Rightarrow \{ \} \quad (89)
\end{align*}
\]
\[
\begin{align*}
B \vdash \text{shareq}_1 & \Rightarrow \{ \} \quad B \vdash \text{shareq}_2 & \Rightarrow \{ \} \\
B \vdash \text{shareq}_1 \land \text{shareq}_2 & \Rightarrow \{ \} \quad (90)
\end{align*}
\]

Comments:

(88) By the definition of consistency the premise is weaker than
\[ B(\text{longstrid}_1) = \cdots = B(\text{longstrid}_n). \] Two different structures with the same name may be thought of as representing different views. The requirement that \( B \) is consistent forces different views to be consistent.
(89) By the definition of consistency the premise is weaker than \( B(\text{longtycon}_1) = \cdots = B(\text{longtycon}_n) \). A type structure with empty constructor environment may have the same type name as one with a non-empty constructor environment; the former could arise from a type description, and the latter from a datatype description. However, the requirement that \( B \) is consistent will prevent two type structures with constructor environments which have different non-empty domains from sharing the same type name.

**Functor Specifications**

\[
\frac{B \vdash \text{fundesc} \Rightarrow F}{B \vdash \text{functor } \text{fundesc} \Rightarrow F}
\]

\[
B \vdash \Rightarrow \{\}
\]

\[
\frac{B \vdash \text{funspec}_1 \Rightarrow F_1 \quad B + F_1 \vdash \text{funspec}_2 \Rightarrow F_2}{B \vdash \text{funspec}_1 \langle ; \rangle \text{funspec}_2 \Rightarrow F_1 + F_2}
\]

*Comments:*

(91) The second closure restriction of Section 3.6 can be enforced by replacing the \( B \) in the premise by \( B_0 + G \) of \( B \).

**Functor Descriptions**

\[
\frac{B \vdash \text{funsigexp} \Rightarrow \Phi \quad (B \vdash \text{fundesc} \Rightarrow F)}{B \vdash \text{funid } \text{funsigexp} \langle \text{and } \text{fundesc} \rangle \Rightarrow \{ \text{funid } \Rightarrow \Phi \}{+ F}}
\]

**Functor Signature Expressions**

\[
\frac{B \vdash \text{sigexp} \Rightarrow (N)S \quad B \oplus \{ \text{strid } \Rightarrow S \} \vdash \text{sigexp}' \Rightarrow (N')S'}{B \vdash (\text{strid } : \text{sigexp}) : \text{sigexp}' \Rightarrow (N)(S,(N')S')}
\]

*Comment:* The signatures \((N)S\) and \((N')S'\) are equality-principal and type-explicit, see rule 65.

**Functor Declarations**

\[
\frac{B \vdash \text{funbind} \Rightarrow F}{B \vdash \text{functor } \text{funbind} \Rightarrow F}
\]

\[
B \vdash \Rightarrow \{\}
\]
\[
\frac{B \vdash \text{fundec}_1 \Rightarrow F_1 \quad B + F_1 \vdash \text{fundec}_2 \Rightarrow F_2}{B \vdash \text{fundec}_1 \langle \cdot, \rangle \text{fundec}_2 \Rightarrow F_1 + F_2}
\]

Comments:

(96) The third closure restriction of Section 3.6 can be enforced by replacing the \(B\) in the premise by \(B_0 + (G \text{ of } B) + (F \text{ of } B)\).

Functor Bindings

\[
B \vdash \text{funbind} \Rightarrow F
\]

\[
\frac{B \vdash \text{sigexp} \Rightarrow (N)S \quad B \oplus \{ \text{strid} \mapsto S\} \vdash \text{streq} \Rightarrow S'}{\langle B \oplus \{ \text{strid} \mapsto S\} \vdash \text{sigexp}' \Rightarrow \Sigma', \Sigma' \geq S'' \prec S' \rangle \\
N' = \text{names } S'' \setminus ((N \text{ of } B) \cup N) \\
\langle \langle B \vdash \text{funbind} \Rightarrow F \rangle \rangle}
\]

Comment: The requirement that \((N)S\) be equality-principal, implicit in the first premise, forces \((N)S\) to be as general as possible given the sharing constraints in \text{sigexp}. The requirement that \((N)S\) be type-explicit ensures that there is at most one realisation via which an actual argument can match \((N)S\). Since \(\oplus\) is used, any structure name \(m\) and type name \(t\) in \(S\) acts like a constant in the functor body; in particular, it ensures that further names generated during elaboration of the body are distinct from \(m\) and \(t\). The set \(N'\) is chosen such that every name free in \((N)S\) or \((N)(S,(N')S')\) is free in \(B\).

Top-level Declarations

\[
B \vdash \text{strdec} \Rightarrow E \quad \text{imptyvars } E = \emptyset
\]

\[
B \vdash \text{strdec} \Rightarrow \langle \text{names } E, E \rangle \text{ in Basis}
\]

\[
B \vdash \text{sigdec} \Rightarrow G \quad \text{imptyvars } G = \emptyset
\]

\[
B \vdash \text{sigdec} \Rightarrow \langle \text{names } G, G \rangle \text{ in Basis}
\]

\[
B \vdash \text{fundec} \Rightarrow F \quad \text{imptyvars } F = \emptyset
\]

\[
B \vdash \text{fundec} \Rightarrow \langle \text{names } F, F \rangle \text{ in Basis}
\]

Comments:

(100)–(102) The side conditions ensure that no free imperative type variables enter the basis.
5.15 Functor Signature Matching

As pointed out in Section 3.4 on the grammar for Modules, there is no phrase class whose elaboration requires matching one functor signature to another functor signature. But a precise definition of this matching is needed, since a functor \( g \) may only be separately compiled in the presence of specification of any functor \( f \) to which \( g \) refers, and then a real functor \( f \) must match this specification. In the case, then, that \( f \) has been specified by a functor signature

\[
\Phi_1 = (N_1)(S_1, (N_1')S_1')
\]

and that later \( f \) is declared with functor signature

\[
\Phi_2 = (N_2)(S_2, (N_2')S_2')
\]

the following matching rule will be employed:

A functor signature \( \Phi_2 = (N_2)(S_2, (N_2')S_2') \) matches another functor signature, \( \Phi_1 = (N_1)(S_1, (N_1')S_1') \), if there exists a realisation \( \varphi \) such that

1. \( (N_1)S_1 \) matches \( (N_2)S_2 \) via \( \varphi \), and
2. \( \varphi((N_2')S_2') \) matches \( (N_1')S_1' \).

The first condition ensures that the real functor signature \( \Phi_2 \) for \( f \) requires the argument \( \text{strexp} \) of any application \( f(\text{strexp}) \) to have no more sharing, and no more richness, than was predicted by the specified signature \( \Phi_1 \). The second condition ensures that the real functor signature \( \Phi_2 \), instantiated to \( (\varphi S_2, \varphi((N_2')S_2')) \), provides in the result of the application \( f(\text{strexp}) \) no less sharing, and no less richness, than was predicted by the specified signature \( \Phi_1 \).
6 Dynamic Semantics for the Core

6.1 Reduced Syntax

Since types are fully dealt with in the static semantics, the dynamic semantics ignores them. The Core syntax is therefore reduced by the following transformations, for the purpose of the dynamic semantics:

- All explicit type ascriptions “: ty” are omitted, and qualifications “of ty” are omitted from exception bindings.
- Any declaration of the form “type typbind” or “datatype datbind” is replaced by the empty declaration.
- A declaration of the form “abtype datbind with dec end” is replaced by “dec”.
- The Core phrase classes TypBind, DatBind, ConBind, Ty and TyRow are omitted.

6.2 Simple Objects

All objects in the dynamic semantics are built from identifier classes together with the simple object classes shown (with the variables which range over them) in Figure 12.

\[
\begin{align*}
\text{addr} & \in \text{Addr} & \text{addresses} \\
\text{ex} & \in \text{ExName} & \text{exception names} \\
\text{bas} & \in \text{BasVal} & \text{basic values} \\
\text{spe} & \in \text{SVal} & \text{special values} \\
\{\text{FAIL}\} & & \text{failure}
\end{align*}
\]

Figure 12: Simple Semantic Objects

Addr and ExName are infinite sets. BasVal is described below. SVal is the class of values denoted by the special constants SCon. Each integer or real constant denotes a value according to normal mathematical conventions; each string constant denotes a sequence of ASCII characters as explained in Section 2.2. The value denoted by scon is written val(scon). FAIL is the result of a failing attempt to match a value and a pattern. Thus FAIL is neither a value nor an exception, but simply a semantic object used in the rules to express operationally how matching proceeds.

Exception constructors evaluate to exception names, unlike value constructors which simply evaluate to themselves. This is to accommodate the generative
\[ v \in \text{Val} = \{:=\} \cup \text{SVal} \cup \text{BasVal} \cup \text{Con} \]
\[ \cup (\text{Con} \times \text{Val}) \cup \text{ExVal} \]
\[ \cup \text{Record} \cup \text{Addr} \cup \text{Closure} \]
\[ r \in \text{Record} = \text{Lab} \xrightarrow{\text{fn}} \text{Val} \]
\[ e \in \text{ExVal} = \text{ExName} \cup (\text{ExName} \times \text{Val}) \]
\[ [e] \text{ or } p \in \text{Pack} = \text{ExVal} \]
\[ (\text{match}, E, \text{VE}) \in \text{Closure} = \text{Match} \times \text{Env} \times \text{VarEnv} \]
\[ \text{mem} \in \text{Mem} = \text{Addr} \xrightarrow{\text{fin}} \text{Val} \]
\[ \text{ens} \in \text{ExNameSet} = \text{Fin}(\text{ExName}) \]
\[ (\text{mem, ens}) \text{ or } s \in \text{State} = \text{Mem} \times \text{ExNameSet} \]
\[ (\text{SE, VE, EE}) \text{ or } E \in \text{Env} = \text{StrEnv} \times \text{VarEnv} \times \text{ExConEnv} \]
\[ \text{SE} \in \text{StrEnv} = \text{StrId} \xrightarrow{\text{fin}} \text{Env} \]
\[ \text{VE} \in \text{VarEnv} = \text{Var} \xrightarrow{\text{fin}} \text{Val} \]
\[ \text{EE} \in \text{ExConEnv} = \text{ExCon} \xrightarrow{\text{fin}} \text{ExName} \]

Figure 13: Compound Semantic Objects

nature of exception bindings; each evaluation of a declaration of a exception constructor binds it to a new unique name.

6.3 Compound Objects

The compound objects for the dynamic semantics are shown in Figure 13. Many conventions and notations are adopted as in the static semantics; in particular projection, injection and modification all retain their meaning. We generally omit the injection functions taking \text{Con}, \text{Con} \times \text{Val} etc into \text{Val}. For records \( r \in \text{Record} \) however, we write this injection explicitly as “in \text{Val}”; this accords with the fact that there is a separate phrase class \text{ExpRow}, whose members evaluate to records.

We take \( \cup \) to mean disjoint union over semantic object classes. We also understand all the defined object classes to be disjoint. A particular case deserves mention; \text{ExVal} and \text{Pack} (exception values and packets) are isomorphic classes, but the latter class corresponds to exceptions which have been raised, and therefore has different semantic significance from the former, which is just a subclass of values.

Although the same names, e.g. \( E \) for an environment, are used as in the static semantics, the objects denoted are different. This need cause no confusion since the static and dynamic semantics are presented separately. An important point is that structure names \( m \) have no significance at all in the dynamic semantics; this explains why the object class \( \text{Str} = \text{StrName} \times \text{Env} \) is absent here – for the dynamic semantics the concepts \text{structure} and \text{environment} coincide.
6.4 Basic Values

The basic values in BasVal are the values bound to predefined variables. These values are denoted by the identifiers to which they are bound in the initial dynamic basis (see Appendix D), and are as follows:

\[
\begin{align*}
\text{abs} & \quad \text{floor} & \quad \text{real} & \quad \text{sqrt} & \quad \text{sin} & \quad \text{cos} & \quad \text{arctan} & \quad \text{exp} & \quad \text{ln} \\
\text{size} & \quad \text{chr} & \quad \text{ord} & \quad \text{expode} & \quad \text{implode} & \quad \text{div} & \quad \text{mod} \\
- & \quad / & \quad * & \quad + & \quad - & \quad = & \quad <> & \quad < & \quad <= & \quad >= \\
\text{std_in} & \quad \text{std_out} & \quad \text{open_in} & \quad \text{open_out} & \quad \text{close_in} & \quad \text{close_out} & \quad \text{input} & \quad \text{output} & \quad \text{lookahead} & \quad \text{end_of_stream}
\end{align*}
\]

The meaning of basic values (almost all of which are functions) is represented by the function

\[\text{APPLY} : \text{BasVal} \times \text{Val} \to \text{Val} \cup \text{Pack}\]

which is detailed in Appendix D.

6.5 Basic Exceptions

A subset BasExName \(\subset\) ExName of the exception names are bound to predefined exception constructors. These names are denoted by the identifiers to which they are bound in the initial dynamic basis (see Appendix D), and are as follows:

\[
\begin{align*}
\text{Abs} & \quad \text{Ord} & \quad \text{Chr} & \quad \text{Div} & \quad \text{Mod} & \quad \text{Quot} & \quad \text{Prod} \\
\text{Neg} & \quad \text{Sum} & \quad \text{Diff} & \quad \text{Floor} & \quad \text{Sqrt} & \quad \text{Exp} & \quad \text{Ln} \\
\text{Io} & \quad \text{Match} & \quad \text{Bind} & \quad \text{Interrupt}
\end{align*}
\]

The exceptions on the first two lines are raised by corresponding basic functions, where - / * + - correspond respectively to Neg Quot Prod Sum Diff. The details are given in Appendix D. The exception (Io,s), where s is a string, is raised by certain of the basic input/output functions, as detailed in Appendix D. The exceptions Match and Bind are raised upon failure of pattern-matching in evaluating a function fn match or a valbind, as detailed in the rules to follow. Finally, Interrupt is raised by external intervention.

Recall from Section 4.11 that in the context fn match, the match must be irredudant and exhaustive and that the compiler should flag the match if it violates these restrictions. The exception Match can only be raised for a match which is not exhaustive, and has therefore been flagged by the compiler.

For each value binding pat = exp the compiler must issue a report (but still compile) if either pat is not exhaustive or pat contains no variable. This will (on both counts) detect a mistaken declaration like val nil = exp in which the user expects to declare a new variable nil (whereas the language dictates that nil is here a constant pattern, so no variable gets declared). However, these warnings should not be given when the binding is a component of a top-level declaration val valbind; e.g. val x::1 = exp_1 and y = exp_2 is not faulted by the compiler at top level, but may of course generate a Bind exception.
6.6 Closures

The informal understanding of a closure \((\text{match}, E, VE)\) is as follows: when the closure is applied to a value \(v\), \text{match} will be evaluated against \(v\), in the environment \(E\) modified in a special sense by \(VE\). The domain \(\text{Dom} VE\) of this third component contains those function identifiers to be treated recursively in the evaluation. To achieve this effect, the evaluation of \text{match} will take place not in \(E + VE\) but in \(E + \text{Rec} VE\), where

\[
\text{Rec} : \text{VarEnv} \rightarrow \text{VarEnv}
\]

is defined as follows:

- \(\text{Dom(Rec } VE) = \text{Dom } VE\)
- If \(VE(var) \notin \text{Closure}\), then \((\text{Rec } VE)(var) = VE(var)\)
- If \(VE(var) = (\text{match}', E', VE')\) then \((\text{Rec } VE)(var) = (\text{match}', E', VE')\)

The effect is that, before application of \((\text{match}, E, VE)\) to \(v\), the closure values in \(\text{Ran } VE\) are “unrolled” once, to prepare for their possible recursive application during the evaluation of \text{match} upon \(v\).

This device is adopted to ensure that all semantic objects are finite (by controlling the unrolling of recursion). The operator \text{Rec} is invoked in just two places in the semantic rules: in the rule for recursive value bindings of the form “\textbf{rec valbind}”, and in the rule for evaluating an application expression “\texttt{exp at exp}” in the case that \texttt{exp} evaluates to a closure.

6.7 Inference Rules

The semantic rules allow sentences of the form

\[
s, A \vdash \text{phrase} \Rightarrow \text{A}', s'
\]

to be inferred, where \(A\) is usually an environment, \(A'\) is some semantic object and \(s, s'\) are the states before and after the evaluation represented by the sentence. Some hypotheses in rules are not of this form; they are called \textit{side-conditions}. The convention for options is the same as for the Core static semantics.

In most rules the states \(s\) and \(s'\) are omitted from sentences; they are only included for those rules which are directly concerned with the state – either referring to its contents or changing it. When omitted, the convention for restoring them is as follows. If the rule is presented in the form

\[
A_1 \vdash \text{phrase}_1 \Rightarrow A'_1 \quad A_2 \vdash \text{phrase}_2 \Rightarrow A'_2 \quad \cdots
\]

\[
\cdots \quad A_n \vdash \text{phrase}_n \Rightarrow A'_n
\]

\[
A \vdash \text{phrase} \Rightarrow A'
\]

then all \(A_i\) and \(A'_i\) are to be understood as having \(s\) as their state.
then the full form is intended to be
\[
s_0, A_1 \vdash \text{phrase}_1 \Rightarrow A'_1, s'_1 \quad \ldots \quad s_{n-1}, A_n \vdash \text{phrase}_n \Rightarrow A'_n, s'_n
\]
\[
\quad s_0, A \vdash \text{phrase} \Rightarrow A', s_n
\]
(Any side-conditions are left unaltered). Thus the left-to-right order of the hypotheses indicates the order of evaluation. Note that in the case \( n = 0 \), when there are no hypotheses (except possibly side-conditions), we have \( s_n = s_0 \); this implies that the rule causes no side effect. The convention is called the state convention, and must be applied to each version of a rule obtained by inclusion or omission of its options.

A second convention, the exception convention, is adopted to deal with the propagation of exception packets \( p \). For each rule whose full form (ignoring side-conditions) is
\[
\quad s_1, A_1 \vdash \text{phrase}_1 \Rightarrow A'_1, s'_1 \quad \ldots \quad s_n, A_n \vdash \text{phrase}_n \Rightarrow A'_n, s'_n
\]
\[
\quad s, A \vdash \text{phrase} \Rightarrow A', s'_n
\]
and for each \( k, 1 \leq k \leq n \), for which the result \( A'_k \) is not a packet \( p \), an extra rule is added of the form
\[
\quad s_1, A_1 \vdash \text{phrase}_1 \Rightarrow A'_1, s'_1 \quad \ldots \quad s_k, A_k \vdash \text{phrase}_k \Rightarrow p'_k, s'_k
\]
\[
\quad s, A \vdash \text{phrase} \Rightarrow p'_k, s'_k
\]
where \( p'_k \) does not occur in the original rule.\(^2\) This indicates that evaluation of phrases in the hypothesis terminates with the first whose result is a packet (other than one already treated in the rule), and this packet is the result of the phrase in the conclusion.

A third convention is that we allow compound variables (variables built from the variables in Figure 13 and the symbol “/”) to range over unions of semantic objects. For instance the compound variable \( v/p \) ranges over \( \text{Val} \cup \text{Pack} \). We also allow \( x/\text{FAIL} \) to range over \( X \cup \{ \text{FAIL} \} \) where \( x \) ranges over \( X \); furthermore, we extend environment modification to allow for failure as follows:

\[
\text{VE} + \text{FAIL} = \text{FAIL}.
\]

### Atomic Expressions

\[
\begin{align*}
E \vdash \text{at.exp} & \Rightarrow v/p \\
E \vdash \text{scon} & \Rightarrow \text{val}(\text{scon}) \\
E(\text{longvar}) &= v \\
E \vdash \text{longvar} & \Rightarrow v
\end{align*}
\]

\(^2\)There is one exception to the exception convention; no extra rule is added for rule 119 which deals with handlers, since a handler is the only means by which propagation of an exception can be arrested.
6.7 Inference Rules

\[ longcon = \text{strid}_1 \ldots \text{strid}_k \cdot \text{con} \]
\[ E \vdash longcon \Rightarrow \text{con} \]  
(105)

\[ E(\text{longexcon}) = \text{en} \]
\[ E \vdash \text{longexcon} \Rightarrow \text{en} \]  
(106)

\[ \langle E \vdash \text{exprow} \Rightarrow r \rangle \]
\[ E \vdash \{ \langle \text{exprow} \rangle \} \Rightarrow \{ \text{+} \} \text{ in Val} \]  
(107)

\[ E \vdash \text{dec} \Rightarrow E' \quad E + E' \vdash \text{exp} \Rightarrow v \]
\[ E \vdash \text{let dec in exp end} \Rightarrow v \]  
(108)

\[ E \vdash \text{exp} \Rightarrow v \]
\[ E \vdash \langle \text{exp} \rangle \Rightarrow v \]  
(109)

Comments:

(105) Value constructors denote themselves.

(106) Exception constructors are looked up in the exception environment component of \( E \).

Expression Rows

\[ E \vdash \text{exp} \Rightarrow v \quad \langle E \vdash \text{exprow} \Rightarrow r \rangle \]
\[ E \vdash \text{lab} = \text{exp} \langle \ , \ \text{exprow} \rangle \Rightarrow \{ \text{lab} \mapsto v \} \langle + \rangle \]  
(110)

Comment: We may think of components as being evaluated from left to right, because of the state and exception conventions.

Expressions

\[ E \vdash \text{atexp} \Rightarrow v \]
\[ E \vdash \text{atexp} \Rightarrow v \]  
(111)

\[ E \vdash \text{exp} \Rightarrow \text{con} \quad \text{con} \neq \text{ref} \quad E \vdash \text{atexp} \Rightarrow v \]
\[ E \vdash \text{exp atexp} \Rightarrow (\text{con}, v) \]  
(112)

\[ E \vdash \text{exp} \Rightarrow \text{en} \quad E \vdash \text{atexp} \Rightarrow v \]
\[ E \vdash \text{exp atexp} \Rightarrow (\text{en}, v) \]  
(113)

\[ s, E \vdash \text{exp} \Rightarrow \text{ref}, s' \quad s', E \vdash \text{atexp} \Rightarrow v, s'' \quad a \notin \text{Dom(\text{mem of } s'')} \]
\[ s, E \vdash \text{exp atexp} \Rightarrow a, s'' + \{ a \mapsto v \} \]  
(114)

\[ s, E \vdash \text{exp} \Rightarrow :=, s' \quad s', E \vdash \text{atexp} \Rightarrow \{ 1 \mapsto a, 2 \mapsto v \}, s'' \]
\[ s, E \vdash \text{exp atexp} \Rightarrow \{ \} \text{ in Val, } s'' + \{ a \mapsto v \} \]  
(115)
\[ E \vdash exp \Rightarrow b \quad E \vdash atexp \Rightarrow v \quad \text{APPLY}(b, v) = v' \]

\[ \frac{\quad E \vdash \text{exp atexp} \Rightarrow v'}{\quad} \] \hspace{1cm} (116)

\[ E \vdash \text{exp} \Rightarrow (\text{match}, E', VE) \quad E \vdash atexp \Rightarrow v \]

\[ E' + \text{Rec} \text{VE}, v \vdash \text{match} \Rightarrow v' \]

\[ \frac{\quad E \vdash \text{exp atexp} \Rightarrow v'}{\quad} \] \hspace{1cm} (117)

\[ E \vdash \text{exp} \Rightarrow (\text{match}, E', VE) \quad E \vdash atexp \Rightarrow v \]

\[ E' + \text{Rec} \text{VE}, v \vdash \text{match} \Rightarrow \text{FAIL} \]

\[ E \vdash \text{exp atexp} \Rightarrow [\text{Match}] \] \hspace{1cm} (118)

\[ E \vdash \text{exp} \Rightarrow v \]

\[ E \vdash \text{exp handle match} \Rightarrow v \] \hspace{1cm} (119)

\[ E \vdash \text{exp} \Rightarrow [e] \quad E, e \vdash \text{match} \Rightarrow v \]

\[ E \vdash \text{exp handle match} \Rightarrow v \] \hspace{1cm} (120)

\[ E \vdash \text{exp} \Rightarrow [e] \quad E, e \vdash \text{match} \Rightarrow \text{FAIL} \]

\[ E \vdash \text{exp handle match} \Rightarrow [e] \] \hspace{1cm} (121)

\[ E \vdash \text{exp} \Rightarrow e \]

\[ E \vdash \text{raise exp} \Rightarrow [e] \] \hspace{1cm} (122)

\[ E \vdash \text{fn match} \Rightarrow (\text{match}, E, \{\}) \] \hspace{1cm} (123)

Comments:

(114) The side condition ensures that a new address is chosen. There are no rules concerning disposal of inaccessible addresses ("garbage collection").

(112)–(118) Note that none of the rules for function application has a premise in which the operator evaluates to a constructed value, a record or an address. This is because we are interested in the evaluation of well-typed programs only, and in such programs \text{exp} will always have a functional type.

(119) This is the only rule to which the exception convention does not apply. If the operator evaluates to a packet then rule 120 or rule 121 must be used.

(121) Packets that are not handled by the \text{match} propagate.

(123) The third component of the \text{closure} is empty because the match does not introduce new recursively defined values.
6.7 Inference Rules

**Matches**

\[ E, v \vdash \text{match} \Rightarrow v'/p/\text{FAIL} \]

\[ E, v \vdash \text{mrule} \Rightarrow v' \]

\[ E, v \vdash \text{mrule} \ (\mathbf{match}) \Rightarrow v' \]

(124)

\[ E, v \vdash \text{mrule} \Rightarrow \text{FAIL} \]

(125)

\[ E, v \vdash \text{mrule} \Rightarrow \text{FAIL} \]

(126)

**Comment:** A value \( v \) occurs on the left of the turnstile, in evaluating a \textit{match}. We may think of a \textit{match} as being evaluated against a value; similarly, we may think of a pattern as being evaluated against a value. Alternative match rules are tried from left to right.

**Match Rules**

\[ E, v \vdash \text{pat} \Rightarrow \text{VE} \quad E + \text{VE} \vdash \text{exp} \Rightarrow v' \]

\[ E, v \vdash \text{pat} \Rightarrow \text{exp} \Rightarrow v' \]

(127)

\[ E, v \vdash \text{pat} \Rightarrow \text{FAIL} \]

(128)

**Declarations**

\[ E \vdash \text{valbind} \Rightarrow \text{VE} \]

\[ E \vdash \text{val \ valbind} \Rightarrow \text{VE \ in \ Env} \]

(129)

\[ E \vdash \text{exbind} \Rightarrow \text{EE} \]

\[ E \vdash \text{exception \ exbind} \Rightarrow \text{EE \ in \ Env} \]

(130)

\[ E \vdash \text{dec}_1 \Rightarrow E_1 \quad E + E_1 \vdash \text{dec}_2 \Rightarrow E_2 \]

\[ E \vdash \text{local \ dec}_1 \ \text{in \ dec}_2 \ \text{end} \Rightarrow E_2 \]

(131)

\[ E(\text{longstrid}_1) = E_1 \ldots E(\text{longstrid}_k) = E_k \]

\[ E \vdash \text{open \ longstrid}_1 \ldots \text{longstrid}_n \Rightarrow E_1 + \cdots + E_k \]

(132)

\[ E \vdash \{ \} \Rightarrow \{ \} \ \text{in \ Env} \]

(133)

\[ E \vdash \text{dec}_1 \Rightarrow E_1 \quad E + E_1 \vdash \text{dec}_2 \Rightarrow E_2 \]

\[ E \vdash \text{dec}_1 \ (\mathbf{\&}) \ \text{dec}_2 \Rightarrow E_1 + E_2 \]

(134)
Value Bindings

\[ E \vdash \text{valbind} \Rightarrow VE/p \]

\[
\begin{align*}
E \vdash \text{exp} & \Rightarrow v \\
E, v \vdash \text{pat} & \Rightarrow VE \\
\langle E \vdash \text{valbind} \Rightarrow VE' \rangle
\end{align*}
\]

\[ E \vdash \text{pat} = \text{exp} \langle \text{and valbind} \rangle \Rightarrow VE \langle + VE' \rangle \]  

(135)

\[
\begin{align*}
E \vdash \text{exp} & \Rightarrow v \\
E, v \vdash \text{pat} & \Rightarrow \text{FAIL}
\end{align*}
\]

\[ E \vdash \text{pat} = \text{exp} \langle \text{and valbind} \rangle \Rightarrow \text{[Bind]} \]  

(136)

\[
\begin{align*}
E \vdash \text{valbind} & \Rightarrow VE \\
E \vdash \text{rec valbind} & \Rightarrow \text{Rec VE}
\end{align*}
\]

(137)

Exception Bindings

\[ E \vdash \text{exbind} \Rightarrow EE/p \]

\[
\begin{align*}
en \notin \text{ens of } s & \quad s' = s + \{en\} \\
\langle s', E \vdash \text{exbind} \Rightarrow EE, s'' \rangle
\end{align*}
\]

\[ s, E \vdash \text{excon} \langle \text{and exbind} \rangle \Rightarrow \{\text{excon} \mapsto en\} \langle + EE \rangle, s'' \langle \rangle \]  

(138)

\[
\begin{align*}
E(\text{longexcon}) & = \text{en} \\
\langle E \vdash \text{exbind} \Rightarrow EE \rangle
\end{align*}
\]

\[ E \vdash \text{excon} = \text{longexcon} \langle \text{and exbind} \rangle \Rightarrow \{\text{excon} \mapsto en\} \langle + EE \rangle \]  

(139)

Comments:

(138) The two side conditions ensure that a new exception name is generated and recorded as “used” in subsequent states.

Atomic Patterns

\[ E, v \vdash \text{atpat} \Rightarrow VE/\text{FAIL} \]

\[
\begin{align*}
E, v \vdash \_ & \Rightarrow \{\}
\end{align*}
\]

(140)

\[
\begin{align*}
v & = \text{val(scon)} \\
E, v \vdash \text{scon} & \Rightarrow \{\}
\end{align*}
\]

(141)

\[
\begin{align*}
v \neq \text{val(scon)} \\
E, v \vdash \text{scon} & \Rightarrow \text{FAIL}
\end{align*}
\]

(142)

\[
\begin{align*}
E, v \vdash \text{var} & \Rightarrow \{\text{var} \mapsto v\}
\end{align*}
\]

(143)

\[
\begin{align*}
\text{longcon} = \text{strid}_1 \cdots \text{strid}_k, \text{con} & \quad v = \text{con} \\
E, v \vdash \text{longcon} & \Rightarrow \{\}
\end{align*}
\]

(144)

\[
\begin{align*}
\text{longcon} = \text{strid}_1 \cdots \text{strid}_k, \text{con} & \quad v \neq \text{con} \\
E, v \vdash \text{longcon} & \Rightarrow \text{FAIL}
\end{align*}
\]

(145)

\[
\begin{align*}
E(\text{longexcon}) & = v \\
E, v \vdash \text{longexcon} & \Rightarrow \{\}
\end{align*}
\]

(146)
6.7 Inference Rules

\[
E(\text{longexcon}) \neq v \\
E, v \vdash \text{longexcon} \Rightarrow \text{FAIL} \quad (147)
\]

\[
v = \{\} \langle +r \rangle \text{ in Val} \quad \langle E, r \vdash \text{patrow} \Rightarrow \text{VE/FAIL} \rangle \\
E, v \vdash \{ \langle \text{patrow} \rangle \} \Rightarrow \{\} \langle +\text{VE/FAIL} \rangle \quad (148)
\]

\[
E, v \vdash \text{pat} \Rightarrow \text{VE/FAIL} \\
E, v \vdash \langle \text{pat} \rangle \Rightarrow \text{VE/FAIL} \quad (149)
\]

Comments:

(142),(145),(147) Any evaluation resulting in \text{FAIL} must do so because rule 142, rule 145, rule 147, rule 155, or rule 157 has been applied.

**Pattern Rows**

\[
E, r \vdash \text{patrow} \Rightarrow \text{VE/FAIL} \\
\]

\[
E, r \vdash \ldots \Rightarrow \{\} \quad (150)
\]

\[
E, r(\text{lab}) \vdash \text{pat} \Rightarrow \text{FAIL} \\
E, r \vdash \text{lab} = \text{pat} \langle \ , \ \text{patrow} \rangle \Rightarrow \text{FAIL} \quad (151)
\]

\[
E, r(\text{lab}) \vdash \text{pat} \Rightarrow \text{VE} \quad \langle E, r \vdash \text{patrow} \Rightarrow \text{VE'/FAIL} \rangle \\
E, r \vdash \text{lab} = \text{pat} \langle \ , \ \text{patrow} \rangle \Rightarrow \text{VE}\langle + \ \text{VE'/FAIL} \rangle \quad (152)
\]

Comments:

(151),(152) For well-typed programs \text{lab} will be in the domain of \text{r}.

**Patterns**

\[
E, v \vdash \text{atpat} \Rightarrow \text{VE/FAIL} \\
E, v \vdash \text{atpat} \Rightarrow \text{VE/FAIL} \quad (153)
\]

\[
\text{longcon} = \text{strid}_1 \ldots \text{strid}_k, \text{con} \neq \text{ref} \quad v = (\text{con}, v') \\
E, v \vdash \text{atpat} \Rightarrow \text{VE/FAIL} \\
E, v \vdash \text{longcon atpat} \Rightarrow \text{VE/FAIL} \quad (154)
\]

\[
\text{longcon} = \text{strid}_1 \ldots \text{strid}_k, \text{con} \neq \text{ref} \quad v \notin \{\text{con}\} \times \text{Val} \\
E, v \vdash \text{longcon atpat} \Rightarrow \text{FAIL} \quad (155)
\]

\[
E(\text{longexcon}) = \text{en} \quad v = (\text{en}, v') \\
E, v \vdash \text{atpat} \Rightarrow \text{VE/FAIL} \\
E, v \vdash \text{longexcon atpat} \Rightarrow \text{VE/FAIL} \quad (156)
\]

\[
E(\text{longexcon}) = \text{en} \quad v \notin \{\text{en}\} \times \text{Val} \\
E, v \vdash \text{longexcon atpat} \Rightarrow \text{FAIL} \quad (157)
\]
\[ s(a) = v \quad s, E, v \vdash \text{atpat} \Rightarrow VE/FAIL, s \]
\[ s, E, a \vdash \text{ref pat} \Rightarrow VE/FAIL, s \]
\[ E, v \vdash \text{pat} \Rightarrow VE/FAIL \]
\[ E, v \vdash \text{var as pat} \Rightarrow \{ \text{var} \mapsto v \} + VE/FAIL \]

Comments:

(155),(157) Any evaluation resulting in FAIL must do so because rule 142, rule 145, rule 147, rule 155, or rule 157 has been applied.
7 Dynamic Semantics for Modules

7.1 Reduced Syntax

Since signature expressions are mostly dealt with in the static semantics, the dynamic semantics need only take limited account of them. Unlike types, it cannot ignore them completely; the reason is that an explicit signature ascription plays the role of restricting the “view” of a structure - that is, restricting the domains of its component environments. However, the types and the sharing properties of structures and signatures are irrelevant to dynamic evaluation; the syntax is therefore reduced by the following transformations (in addition to those for the Core), for the purpose of the dynamic semantics of Modules:

- Qualifications “of ty” are omitted from exception descriptions.
- Any specification of the form “type typdesc”, “eqtype typdesc”, “datatype datdesc” or “sharing shared” is replaced by the empty specification.
- The Modules phrase classes TypDesc, DatDesc, ConDesc and SharEq are omitted.

7.2 Compound Objects

The compound objects for the Modules dynamic semantics, extra to those for the Core dynamic semantics, are shown in Figure 14. An interface \( I \in \text{Int} \) represents

\[
(\text{strid} : I, \text{strexpr}(:, I'), B) \in \text{FunctorClosure}
\]

\[
= (\text{StrId} \times \text{Int}) \times (\text{StrExp}(\times \text{Int})) \times \text{Basis}
\]

\[
(IE, \text{vars}, \text{excons}) \text{ or } I \in \text{Int} = \text{IntEnv} \times \text{Fin(Var)} \times \text{Fin(ExCon)}
\]

\[
IE \in \text{IntEnv} = \text{StrId} \mapsto \text{Int}
\]

\[
G \in \text{SigEnv} = \text{SigId} \mapsto \text{Int}
\]

\[
F \in \text{FunEnv} = \text{FunId} \mapsto \text{FunctorClosure}
\]

\[
(F, G, E) \text{ or } B \in \text{Basis} = \text{FunEnv} \times \text{SigEnv} \times \text{Env}
\]

\[
(G, IE) \text{ or } IB \in \text{IntBasis} = \text{SigEnv} \times \text{IntEnv}
\]

Figure 14: Compound Semantic Objects

a “view” of a structure. Specifications and signature expressions will evaluate to interfaces; moreover, during the evaluation of a specification or signature expression, structures (to which a specification or signature expression may refer via “open”) are represented only by their interfaces. To extract an interface from a dynamic environment we define the operation

\[
\text{Inter} : \text{Env} \rightarrow \text{Int}
\]
as follows:
$$\text{Inter}(SE, VE, EE) = (IE, \text{Dom}VE, \text{Dom}EE)$$
where
$$IE = \{\text{strid} \mapsto \text{Inter}E \; ; \; SE(\text{strid}) = E\}.$$  
An interface basis $IB = (G, IE)$ is that part of a basis needed to evaluate signature expressions and specifications. The function Inter is extended to create an interface basis from a basis $B$ as follows:
$$\text{Inter}(F, G, E) = (G, IE \circ (\text{Inter} E))$$
A further operation
$$\downarrow: \text{Env} \times \text{Int} \rightarrow \text{Env}$$
is required, to cut down an environment $E$ to a given interface $I$, representing the effect of an explicit signature ascription. It is defined as follows:
$$(SE, VE, EE) \downarrow (IE, \text{vars}, \text{excons}) = (SE', VE', EE')$$
where
$$SE' = \{\text{strid} \mapsto E \downarrow I \; ; \; SE(\text{strid}) = E \text{ and } IE(\text{strid}) = I\}$$
and (taking $\downarrow$ now to mean restriction of a function domain)
$$VE' = VE \downarrow \text{vars}, \; EE' = EE \downarrow \text{excons}.$$  

It is important to note that an interface is also a projection of the static value $\Sigma$ of a signature expression; it is obtained by omitting structure names $m$ and type environments $TE$, and replacing each variable environment $VE$ and each exception environment $EE$ by its domain. Thus in an implementation interfaces would naturally be obtained from the static elaboration; we choose to give separate rules here for obtaining them in the dynamic semantics since we wish to maintain our separation of the static and dynamic semantics, for reasons of presentation.

### 7.3 Inference Rules

The semantic rules allow sentences of the form
$$s, A \vdash \text{phrase} \Rightarrow A', s'$$
to be inferred, where $A$ is either a basis or an interface basis or empty, $A'$ is some semantic object and $s, s'$ are the states before and after the evaluation represented by the sentence. Some hypotheses in rules are not of this form; they are called side-conditions. The convention for options is the same as for the Core static semantics.

The state and exception conventions are adopted as in the Core dynamic semantics. However, it may be shown that the only Modules phrases whose evaluation may cause a side-effect or generate an exception packet are of the form $\text{strexp}$, $\text{strdec}$, $\text{strbind}$ or $\text{topdec}$. 

7.3 Inference Rules

Structure Expressions

\[ B \vdash stdec \Rightarrow E \]
\[ B \vdash \text{struct } stdec \text{ end } \Rightarrow E \] (160)

\[ B(\text{longstrid}) = E \]
\[ B \vdash \text{longstrid } \Rightarrow E \] (161)

\[ B(\text{funid}) = (\text{strid } : I, stexp'(: I'), B') \]
\[ B \vdash stexp \Rightarrow E \]
\[ B' + \{ \text{strid } \mapsto E \downarrow I \} \vdash stexp' \Rightarrow E' \]
\[ B \vdash \text{funid } (\text{stexp} ) \Rightarrow E'(\downarrow I') \] (162)

\[ B \vdash stdec \Rightarrow E \]
\[ B + E \vdash stexp \Rightarrow E' \]
\[ B \vdash \text{let } stdec \text{ in } stexp \text{ end } \Rightarrow E' \] (163)

Comments:

(162) Before the evaluation of the functor body \( stexp' \), the actual argument \( E \) is cut down by the formal parameter interface \( I \), so that any opening of \( \text{strid} \) resulting from the evaluation of \( stexp' \) will produce no more components than anticipated during the static elaboration.

Structure-level Declarations

\[ B \vdash stdec \Rightarrow E/p \]

\[ E \text{ of } B \vdash \text{dec } \Rightarrow E' \]
\[ B \vdash \text{dec } \Rightarrow E' \] (164)

\[ B \vdash \text{strbind } \Rightarrow SE \]
\[ B \vdash \text{structure } \text{strbind } \Rightarrow SE \text{ in Env} \] (165)

\[ B \vdash stdec_1 \Rightarrow E_1 \]
\[ B + E_1 \vdash stdec_2 \Rightarrow E_2 \]
\[ B \vdash \text{local } stdec_1 \text{ in } stdec_2 \text{ end } \Rightarrow E_2 \] (166)

\[ B \vdash \Rightarrow \{ \} \text{ in Env} \] (167)

\[ B \vdash stdec_1 \Rightarrow E_1 \]
\[ B + E_1 \vdash stdec_2 \Rightarrow E_2 \]
\[ B \vdash stdec_1 \langle ; \rangle \text{ stdec}_2 \Rightarrow E_1 + E_2 \] (168)
Structure Bindings

\[ B \vdash \text{strbind} \Rightarrow SE/p \]

\[ B \vdash \text{stexp} \Rightarrow E \quad \langle \text{Inter } B \vdash \text{sigexp} \Rightarrow I \rangle \]

\[ \langle B \vdash \text{strbind} \Rightarrow SE \rangle \]

\[ B \vdash \text{strid} \langle \cdot \text{ sigexp} \rangle = \text{stexp} \langle \langle \text{and strbind} \rangle \rangle \Rightarrow \{ \text{strid} \mapsto E \langle \downarrow I \rangle \} \langle \langle + SE \rangle \rangle \]  \hspace{1cm} (169)

\text{Comment: As in the static semantics, when present, \text{sigexp} constrains the “view” of the structure. The restriction must be done in the dynamic semantics to ensure that any dynamic opening of the structure produces no more components than anticipated during the static elaboration.}

Signature Expressions

\[ IB \vdash \text{spec} \Rightarrow I \]

\[ IB \vdash \text{spec end} \Rightarrow I \]  \hspace{1cm} (170)

\[ IB(\text{sigid}) = I \]

\[ IB \vdash \text{sigid} \Rightarrow I \]  \hspace{1cm} (171)

Signature Declarations

\[ IB \vdash \text{sigdec} \Rightarrow G \]

\[ IB \vdash \text{sigbind} \Rightarrow G \]

\[ IB \vdash \text{signature sigbind} \Rightarrow G \]  \hspace{1cm} (172)

\[ IB \vdash \Rightarrow \{ \} \]  \hspace{1cm} (173)

\[ IB \vdash \text{sigdec}_1 \Rightarrow G_1 \quad IB + G_1 \vdash \text{sigdec}_2 \Rightarrow G_2 \]

\[ IB \vdash \text{sigdec}_1 \langle ; \rangle \text{ sigdec}_2 \Rightarrow G_1 + G_2 \]  \hspace{1cm} (174)

Signature Bindings

\[ IB \vdash \text{sigbind} \Rightarrow G \]

\[ IB \vdash \text{spec} \Rightarrow I \]

\[ \vdash \text{valdesc} \Rightarrow \text{vars} \]

\[ IB \vdash \text{val valdesc} \Rightarrow \text{vars in Int} \]  \hspace{1cm} (176)

\[ \vdash \text{exdesc} \Rightarrow \text{excons} \]

\[ IB \vdash \text{exception exdesc} \Rightarrow \text{excons in Int} \]  \hspace{1cm} (177)
7.3 Inference Rules

\[ IB \vdash \text{strdesc} \Rightarrow IE \]
\[ \text{IB} \vdash \text{structure strdesc} \Rightarrow IE \text{ in Int} \] (178)
\[ IB \vdash \text{spec}_1 \Rightarrow I_1 \quad IB + IE \text{ of } I_1 \vdash \text{spec}_2 \Rightarrow I_2 \]
\[ IB \vdash \text{local spec}_1 \text{ in spec}_2 \text{ end} \Rightarrow I_2 \] (179)
\[ IB(\text{longstrid}_1) = I_1 \quad \cdots \quad IB(\text{longstrid}_n) = I_n \]
\[ IB \vdash \text{open longstrid}_1 \cdots \text{longstrid}_n \Rightarrow I_1 + \cdots + I_n \] (180)
\[ IB(\text{sigid}_1) = I_1 \quad \cdots \quad IB(\text{sigid}_n) = I_n \]
\[ IB \vdash \text{include sigid}_1 \cdots \text{sigid}_n \Rightarrow I_1 + \cdots + I_n \] (181)
\[ IB \vdash \Rightarrow \{\} \text{ in Int} \]
\[ IB \vdash \text{spec}_1 \Rightarrow I_1 \quad IB + IE \text{ of } I_1 \vdash \text{spec}_2 \Rightarrow I_2 \]
\[ IB \vdash \text{spec}_1 \langle i \rangle \text{ spec}_2 \Rightarrow I_1 + I_2 \] (182)

Comments:

(179),(183) Note that varof \(I_1\) and exconof \(I_1\) are not needed for the evaluation of spec\(_2\).

Value Descriptions

\[ \vdash \text{valdesc} \Rightarrow \text{vars} \]
\[ \langle \vdash \text{valdesc} \Rightarrow \text{vars} \rangle \]
\[ \vdash \text{var} \langle \text{and valdesc} \rangle \Rightarrow \{\text{var}\} \langle \cup \text{vars} \rangle \] (184)

Exception Descriptions

\[ \vdash \text{exdesc} \Rightarrow \text{excons} \]
\[ \langle \vdash \text{exdesc} \Rightarrow \text{excons} \rangle \]
\[ \vdash \text{excon} \langle \text{exdesc} \rangle \Rightarrow \{\text{excon}\} \langle \cup \text{excons} \rangle \] (185)

Structure Descriptions

\[ IB \vdash \text{strdesc} \Rightarrow IE \]
\[ IB \vdash \text{sigexp} \Rightarrow I \quad \langle IB \vdash \text{strdesc} \Rightarrow IE \rangle \]
\[ IB \vdash \text{strid} : \text{sigexp} \langle \text{and strdesc} \rangle \Rightarrow \{\text{strid} \mapsto I\} \langle + IE \rangle \] (186)

Functor Bindings

\[ B \vdash \text{funbind} \Rightarrow F \]
\[ \text{Inter } B \vdash \text{sigexp} \Rightarrow I \quad \langle \text{Inter } B + \{\text{strid} \mapsto I\} \vdash \text{sigexp}' \Rightarrow I' \rangle \]
\[ \langle B \vdash \text{funbind} \Rightarrow F \rangle \]
\[ B \vdash \text{funid} \langle \text{strid} : \text{sigexp} \rangle \langle : \text{sigexp}' \rangle = \text{strexp} \langle \langle \text{and funbind} \rangle \rangle \Rightarrow \{\text{funid} \mapsto (\text{strid} : I, \text{strexp} : I', B)\} \langle + F \rangle \] (187)
Functor Declarations

\[ B \vdash \text{funbind} \Rightarrow F \]
\[ B \vdash \text{functor funbind} \Rightarrow F \]  
\[ B \vdash \phi \Rightarrow \{ \} \]  
\[ B \vdash \text{fundec}_1 \Rightarrow F_1 \quad B + F_1 \vdash \text{fundec}_2 \Rightarrow F_2 \]
\[ B \vdash \text{fundec}_1 \langle ; \rangle \text{fundec}_2 \Rightarrow F_1 + F_2 \]  

Top-level Declarations

\[ B \vdash \text{topdec} \Rightarrow B'/p \]
\[ B \vdash \text{strdec} \Rightarrow E \]
\[ B \vdash \text{strdec} \Rightarrow E \text{ in Basis} \]  
\[ \text{Inter} B \vdash \text{sigdec} \Rightarrow G \]
\[ B \vdash \text{sigdec} \Rightarrow G \text{ in Basis} \]  
\[ B \vdash \text{fundec} \Rightarrow F \]
\[ B \vdash \text{fundec} \Rightarrow F \text{ in Basis} \]
8 Programs

The phrase class Program of programs is defined as follows

\[
\text{program ::= topdec ; \langle program \rangle}
\]

Hitherto, the semantic rules have not exposed the interactive nature of the language. During an ML session the user can type in a phrase, more precisely a phrase of the form topdec as defined in Figure 8, page 14. Upon the following semicolon, the machine will then attempt to parse, elaborate and evaluate the phrase returning either a result or, if any of the phases fail, an error message. The outcome is significant for what the user subsequently types, so we need to answer questions such as: if the elaboration of a top-level declaration succeeds, but its evaluation fails, then does the result of the elaboration get recorded in the static basis?

In practice, ML implementations may provide a directive as a form of top-level declaration for including programs from files rather than directly from the terminal. In case a file consists of a sequence of top-level declarations (separated by semicolons) and the machine detects an error in one of these, it is probably sensible to abort the execution of the directive. Rather than introducing a distinction between, say, batch programs and interactive programs, we shall tacitly regard all programs as interactive, and leave to implementers to clarify how the inclusion of files, if provided, affects the updating of the static and dynamic basis. Moreover, we shall focus on elaboration and evaluation and leave the handling of parse errors to implementers (since it naturally depends on the kind of parser being employed). Hence, in this section the execution of a program means the combined elaboration and evaluation of the program.

So far, for simplicity, we have used the same notation \( B \) to stand for both a static and a dynamic basis, and this has been possible because we have never needed to discuss static and dynamic semantics at the same time. In giving the semantics of programs, however, let us rename as StaticBasis the class Basis defined in the static semantics of modules, Section 5.1, and let us use \( B_{\text{STAT}} \) to range over StaticBasis. Similarly, let us rename as DynamicBasis the class Basis defined in the dynamic semantics of modules, Section 7.2, and let us use \( B_{\text{DYN}} \) to range over DynamicBasis. We now define

\[
B \text{ or } (B_{\text{STAT}}, B_{\text{DYN}}) \in \text{Basis} = \text{StaticBasis} \times \text{DynamicBasis}.
\]

Further, we shall use \( \vdash_{\text{STAT}} \) for elaboration as defined in Section 5, and \( \vdash_{\text{DYN}} \) for evaluation as defined in Section 7. Then \( \vdash \) will be reserved for the execution of programs, which thus is expressed by a sentence of the form

\[
s, B \vdash \text{program} \Rightarrow B', s'
\]

This may be read as follows: starting in basis \( B \) with state \( s \) the execution of program results in a basis \( B' \) and a state \( s' \).
It must be understood that executing a program never results in an exception. If the evaluation of a topdec yields an exception (for instance because of a raise expression or external intervention) then the result of executing the program “topdec ;” is the original basis together with the state which is in force when the exception is generated. In particular, the exception convention of Section 6.7 is not applicable to the ensuing rules.

We represent the non-elaboration of a top-level declaration by \( \cdots \vdash_{\text{STAT}} \text{topdec} \not\Rightarrow \). (This covers also the case in which a user interrupts the elaboration.)

Programs

\[
\begin{align*}
B_{\text{STAT}} & \vdash_{\text{STAT}} \text{topdec} \not\Rightarrow \quad \langle s, B \vdash \text{program} \Rightarrow B', s' \rangle \\
\hline
s, B \vdash \text{topdec} ; \langle \text{program} \rangle \Rightarrow B', s' 
\end{align*}
\]

(194)

\[
\begin{align*}
B_{\text{STAT}} & \vdash_{\text{STAT}} \text{topdec} \Rightarrow B_{\text{STAT}}^{(1)} \\
s, B_{\text{DYN}} & \vdash_{\text{DYN}} \text{topdec} \Rightarrow p, s' \\
\hline
s, B \vdash \text{topdec} ; \langle \text{program} \rangle \Rightarrow B', s' 
\end{align*}
\]

(195)

\[
\begin{align*}
B_{\text{STAT}} & \vdash_{\text{STAT}} \text{topdec} \Rightarrow B_{\text{STAT}}^{(1)} \\
s, B_{\text{DYN}} & \vdash_{\text{DYN}} \text{topdec} \Rightarrow B_{\text{DYN}}^{(1)}, s' \\
B' & = B \oplus (B_{\text{STAT}}^{(1)}, B_{\text{DYN}}^{(1)}) \\
\hline
s, B \vdash \text{topdec} ; \langle \text{program} \rangle \Rightarrow B', s' 
\end{align*}
\]

(196)

Comments:

(194) A failing elaboration has no effect whatever.

(195) An evaluation which yields an exception nullifies the change in the static basis, but does not nullify side-effects on the state which may have occurred before the exception was raised.

Core language Programs

A program is called a core language program if it can be parsed in the reduced grammar defined as follows:

1. Replace the definition of top-level declarations by

   \[
   \text{topdec} ::= \text{strdec}
   \]

2. Replace the definition of structure-level declarations by

   \[
   \text{strdec} ::= \text{dec}
   \]
3. Omit the open declaration from the syntax class of declarations $dec$

4. Restrict the long identifier classes to identifiers, i.e. omit qualified identifiers.

This means that several components of a basis, for example the signature and functor environments, are irrelevant to the execution of a core language program.
A Appendix: Derived Forms

Several derived grammatical forms are provided in the Core; they are presented in Figures 15, 16 and 17. Each derived form is given with its equivalent form. Thus, each row of the tables should be considered as a rewriting rule

\[ \text{Derived form} \implies \text{Equivalent form} \]

and these rules may be applied repeatedly to a phrase until it is transformed into a phrase of the bare language. See Appendix B for the full Core grammar, including all the derived forms.

In the derived forms for tuples, in terms of records, we use \( n \) to mean the ML numeral which stands for the natural number \( n \).

Note that a new phrase class \( \text{FvalBind} \) of function-value bindings is introduced, accompanied by a new declaration form \( \text{fun fvalbind} \). The mixed forms \( \text{val rec fvalbind} \), \( \text{val fvalbind} \) and \( \text{fun valbind} \) are not allowed — though the first form arises during translation into the bare language.

The following notes refer to Figure 17:

- There is a version of the derived form for function-value binding which allows the function identifier to be infixed; see Figure 20 in Appendix B.

- In the two forms involving \( \text{withtype} \), the identifiers bound by \( \text{datbind} \) and by \( \text{typbind} \) must be distinct. Then the transformed binding \( \text{datbind}' \) in the equivalent form is obtained from \( \text{datbind} \) by expanding out all the definitions made by \( \text{typbind} \). More precisely, if \( \text{typbind} \) is

\[
\text{tyvarseq}_1 \text{ tycon}_1 \equiv \text{ty}_1 \quad \text{and} \quad \cdots \quad \text{and} \quad \text{tyvarseq}_n \text{ tycon}_n \equiv \text{ty}_n
\]

then \( \text{datbind}' \) is the result of simultaneous replacement (in \( \text{datbind} \)) of every type expression \( \text{tyseq}_i \text{ tycon}_i \) \((1 \leq i \leq n)\) by the corresponding defining expression

\[
\text{ty}_i\{\text{tyseq}_i/\text{tyvarseq}_i\}
\]

Figure 18 shows derived forms for functors. They allow functors to take, say, a single type or value as a parameter, in cases where it would seem clumsy to "wrap up" the argument as a structure expression. These forms are currently more experimental than the bare syntax of modules, but we recommend implementers to include them so that they can be tested in practice. In the derived forms for functor bindings and functor signature expressions, \( \text{strid} \) is a new structure identifier and the form of \( \text{sigexp}' \) depends on the form of \( \text{sigexp} \) as follows. If \( \text{sigexp} \) is simply a signature identifier \( \text{sigid} \), then \( \text{sigexp}' \) is also \( \text{sigid} \); otherwise \( \text{sigexp} \) must take the form \( \text{sig spec}_1 \text{ end} \), and then \( \text{sigexp}' \) is \( \text{sig local open strid in spec}_1 \text{ end end} \).
### Derived Form | Equivalent Form

**Expressions** \( \text{exp} \)

| \( () \) | \{ \} |
| \( (\text{exp}_1, \ldots, \text{exp}_n) \) | \{i=\text{exp}_1, \ldots, \text{m}=\text{exp}_n\} |
| \# \text{lab} | fn \{\text{lab}=\text{var}, \ldots\} => \text{var} |
| \text{case} \ \text{exp} \ \text{of} \ \text{match} | (fn \ \text{match})(\text{exp}) |
| \text{if} \ \text{exp}_1 \ \text{then} \ \text{exp}_2 \ \text{else} \ \text{exp}_3 | \text{case} \ \text{exp}_1 \ \text{of} \ \text{true} => \text{exp}_2 \ |
| \text{if} \ \text{exp}_1 \ \text{then} \ \text{true} \ \text{else} \ \text{exp}_2 | \text{false} => \text{exp}_3 |
| \text{exp}_1 \ \text{orelse} \ \text{exp}_2 | \text{if} \ \text{exp}_1 \ \text{then} \ \text{true} \ \text{else} \ \text{exp}_2 |
| \text{exp}_1 \ \text{andalso} \ \text{exp}_2 | \text{if} \ \text{exp}_1 \ \text{then} \ \text{exp}_2 \ \text{else} \ \text{false} |
| \( (\text{exp}_1; \ldots; \text{exp}_n; \text{exp}) \) | \text{case} \ \text{exp}_1 \ \text{of} \ \_ => \ |
| \( \ldots \)| case \ \text{exp}_n \ \text{of} \ \_ => \ \text{exp} |
| \text{let} \ \text{dec} \ \text{in} \ | \text{let} \ \text{dec} \ \text{in} \ |
| \text{exp}_1; \ldots; \text{exp}_n \ \text{end} | (\text{exp}_1; \ldots; \text{exp}_n) \ \text{end} |
| \text{while} \ \text{exp}_1 \ \text{do} \ \text{exp}_2 | \text{let} \ \text{val} \ \text{rec} \ \text{var} = fn() => \ |
| \text{if} \ \text{exp}_1 \ \text{then} \ (\text{exp}_2; \text{var}) \ \text{else} () | \text{in} \ \text{var}() \ \text{end} |
| \[\text{exp}_1, \ldots, \text{exp}_n\] | \text{exp}_1 :: \ldots :: \text{exp}_n :: \text{nil} |

\( n \geq 2 \) \hspace{1cm} \text{(\text{var new})}

Figure 15: Derived forms of Expressions

### Derived Form | Equivalent Form

**Patterns** \( \text{pat} \)

| \( () \) | \{ \} |
| \( (\text{pat}_1, \ldots, \text{pat}_n) \) | \{i=\text{pat}_1, \ldots, \text{m}=\text{pat}_n\} |
| \[\text{pat}_1, \ldots, \text{pat}_n\] | \text{pat}_1 :: \ldots :: \text{pat}_n :: \text{nil} |

\( n \geq 2 \) \hspace{1cm} \text{(\text{patrow})}

**Pattern Rows** \( \text{patrow} \)

| \id{\text{:ty}} \ \text{\{as pat\} (, \ \text{patrow})} | \id = \id{\text{:ty}} \ \text{\{as pat\} (, \ \text{patrow})} |

**Type Expressions** \( \text{ty} \)

| \( \text{ty}_1 * \ldots * \text{ty}_n \) | \{1:\text{ty}_1, \ldots, \text{m}:\text{ty}_n\} |

\( n \geq 2 \)

Figure 16: Derived forms of Patterns and Type Expressions
## Derived Form

<table>
<thead>
<tr>
<th><strong>Function-value Bindings</strong> $fvalbind$</th>
<th><strong>Equivalent Form</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle op \rangle var \ atm_{11} \ldots \atm_{1n} (: ty) = exp_1$</td>
<td>$\langle op \rangle var = \text{fn} \ var_1 =&gt; \ldots \text{fn} \ var_n =&gt;$</td>
</tr>
<tr>
<td>$\langle op \rangle var \ atm_{21} \ldots \atm_{2n} (: ty) = exp_2$</td>
<td>case $(\text{var}<em>1, \ldots, \text{var}<em>n)$ of $\langle atm</em>{11}, \ldots, atm</em>{1n} \rangle =&gt; \text{exp}_1 (: ty)$</td>
</tr>
<tr>
<td>... ...</td>
<td>$\langle atm_{21}, \ldots, atm_{2n} \rangle =&gt; \text{exp}_2 (: ty)$</td>
</tr>
<tr>
<td>$\langle op \rangle var \ atm_{m1} \ldots \atm_{mn} (: ty) = exp_m$</td>
<td>$\langle atm_{m1}, \ldots, atm_{mn} \rangle =&gt; \text{exp}_m (: ty)$</td>
</tr>
<tr>
<td>(and $fvalbind$)</td>
<td>(and $fvalbind$)</td>
</tr>
</tbody>
</table>

$(m, n \geq 1; \text{var}_1, \ldots, \text{var}_n$ distinct and new)

### Declarations $dec$

<table>
<thead>
<tr>
<th>fun $fvalbind$</th>
<th>val rec $fvalbind$</th>
</tr>
</thead>
<tbody>
<tr>
<td>datatype $datbind$ withtype $typbind$</td>
<td>datatype $datbind'$ ; type $typbind$</td>
</tr>
<tr>
<td>abstype $datbind$ withtype $typbind$ with $dec$ end</td>
<td>abstype $datbind'$ with type $typbind$ ; $dec$ end</td>
</tr>
</tbody>
</table>

(see note in text concerning $datbind'$)

---

## Derived Form

<table>
<thead>
<tr>
<th><strong>Structure Expressions</strong> $stexp$</th>
<th><strong>Equivalent Form</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{funid} (\text{strdec})$</td>
<td>$\text{funid} (\text{struct strdec end})$</td>
</tr>
</tbody>
</table>

### Functor Bindings $funbind$

<table>
<thead>
<tr>
<th>$\text{funid} (\text{spec}) (: \text{sigexp}) = \text{stexp} (\text{and funbind})$</th>
<th>$\text{funid} (\text{strid} : \text{sig spec end}) (: \text{sigexp}') =$</th>
</tr>
</thead>
</table>
| let open $\text{strid}$ in $\text{stexp}$ end (and $\text{funbind}$) | $(\text{strid new};$ see note in text concerning $\text{sigexp}'$)

### Functor Signature Expressions $funsigexp$

<table>
<thead>
<tr>
<th>(spec) : sigexp</th>
<th>(strid : sig spec end) : sigexp'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \text{strid new};$ see note in text concerning $\text{sigexp}'$</td>
<td></td>
</tr>
</tbody>
</table>

### Top-level Declarations $topdec$

| $\text{exp}$ | val it = $\text{exp}$ |

---

Figure 17: Derived forms of Function-value Bindings and Declarations

Figure 18: Derived forms of Functors and Top-level Declarations
B Appendix: Full Grammar

The full grammar of programs is exactly as given at the start of Section 8.

The full grammar of Modules consists of the grammar of Figures 5–8 in Section 3, together with the derived forms of Figure 18 in Appendix A.

The remainder of this Appendix is devoted to the full grammar of the Core. Roughly, it consists of the grammar of Section 2 augmented by the derived forms of Appendix A. But there is a further difference: two additional subclasses of the phrase class Exp are introduced, namely AppExp (application expressions) and InfExp (infix expressions). The inclusion relation among the four classes is as follows:

\[ \text{AtExp} \subset \text{AppExp} \subset \text{InfExp} \subset \text{Exp} \]

The effect is that certain phrases, such as \(2 + \text{while} \cdots \text{do} \cdots\), are now disallowed.

The grammatical rules are displayed in Figures 19, 20, 21 and 22. The grammatical conventions are exactly as in Section 2, namely:

- The brackets \(\langle \rangle\) enclose optional phrases.
- For any syntax class \(X\) (over which \(x\) ranges) we define the syntax class \(X_{\text{seq}}\) (over which \(x_{\text{seq}}\) ranges) as follows:

\[
x_{\text{seq}} ::= x \quad \text{(singleton sequence)}
\]

\[
x_{\text{seq}} ::= (x_1, \ldots, x_n) \quad \text{(sequence, } n \geq 1)\]

(Note that the "\(\cdots\)" used here, a meta-symbol indicating syntactic repetition, must not be confused with "\(\ldots\)" which is a reserved word of the language.)

- Alternative forms for each phrase class are in order of decreasing precedence. This precedence resolves ambiguity in parsing in the following way. Suppose that a phrase class — we take \(\text{exp}\) as an example — has two alternative forms \(F_1\) and \(F_2\), such that \(F_1\) ends with an \(\text{exp}\) and \(F_2\) starts with an \(\text{exp}\). A specific case is

\[
F_1: \quad \text{if } \text{exp}_1 \text{ then } \text{exp}_2 \text{ else } \text{exp}_3
\]

\[
F_2: \quad \text{exp} \text{ handle } \text{match}
\]

It will be enough to see how ambiguity is resolved in this specific case.

Suppose that the lexical sequence

\[
\cdots \text{if } \cdots \text{then } \cdots \text{else } \text{exp} \text{ handle } \cdots
\]

is to be parsed, where \(\text{exp}\) stands for a lexical sequence which is already determined as a subphrase (if necessary by applying the precedence rule).
Then the higher precedence of \( F_2 \) (in this case) dictates that \( exp \) associates to the right, i.e. that the correct parse takes the form

\[
\cdots \text{if} \ \cdots \ \text{then} \ \cdots \ \text{else} \ (exp \ \text{handle} \ \cdots) \ \cdots
\]

not the form

\[
\cdots (\cdots \text{if} \ \cdots \ \text{then} \ \cdots \ \text{else} \ exp) \ \text{handle} \ \cdots \ \cdots
\]

Note particularly that the use of precedence does not decrease the class of admissible phrases; it merely rejects alternative ways of parsing certain phrases. In particular, the purpose is not to prevent a phrase, which is an instance of a form with higher precedence, having a constituent which is an instance of a form with lower precedence. Thus for example

\[
\text{if} \ \cdots \ \text{then while} \ \cdots \ \text{do} \ \cdots \ \text{else while} \ \cdots \ \text{do} \ \cdots
\]

is quite admissible, and will be parsed as

\[
\text{if} \ \cdots \ \text{then} \ (\text{while} \ \cdots \ \text{do} \ \cdots) \ \text{else} \ (\text{while} \ \cdots \ \text{do} \ \cdots)
\]

- L (resp. R) means left (resp. right) association.
- The syntax of types binds more tightly than that of expressions.
- Each iterated construct (e.g. \textit{match}, \cdots) extends as far right as possible; thus, parentheses may be needed around an expression which terminates with a match, e.g. \textit{“in match”}, if this occurs within a larger match.
\begin{verbatim}
\textit{atexp} ::= \textit{scon} \hspace{1cm} \text{special constant}
\langle \text{op} \rangle \textit{longvar} \hspace{1cm} \text{value variable}
\langle \text{op} \rangle \textit{longcon} \hspace{1cm} \text{value constructor}
\langle \text{op} \rangle \textit{longexcon} \hspace{1cm} \text{exception constructor}
\{ \langle expr \rangle \} \hspace{1cm} \text{record}
# \textit{lab} \hspace{1cm} \text{record selector}
() \hspace{1cm} \text{0-tuple}
(\textit{exp}_1 , \ldots , \textit{exp}_n) \hspace{1cm} n\text{-tuple, } n \geq 2
[\textit{exp}_1 , \ldots , \textit{exp}_n] \hspace{1cm} \text{list, } n \geq 0
(\textit{exp}_1 ; \ldots ; \textit{exp}_n) \hspace{1cm} \text{sequence, } n \geq 2
\textit{let dec in} \textit{exp}_1 ; \ldots ; \textit{exp}_n \textit{end} \hspace{1cm} \text{local declaration, } n \geq 1

\textit{exprw} ::= \textit{lab} = \textit{exp} \langle , \, \textit{exprw} \rangle \hspace{1cm} \text{expression row}
\textit{appexp} ::= \textit{atexp} \hspace{1cm} \text{application expression}
\textit{appexp atexp} \hspace{1cm} \text{application expression}
\textit{infixp} ::= \textit{appexp} \hspace{1cm} \text{infix expression}
in\textit{fexp}_1 \textit{id infexp}_2 \hspace{1cm} \text{infix expression}
\textit{exp} ::= \textit{inexp} \hspace{1cm} \text{typed (L)}
\textit{exp} : \textit{ty} \hspace{1cm} \text{conjunction}
\textit{exp}_1 \textit{andalso} \textit{exp}_2 \hspace{1cm} \text{disjunction}
\textit{exp}_1 \textit{oelse} \textit{exp}_2 \hspace{1cm} \text{handle exception}
\textit{exp handle} \textit{match} \hspace{1cm} \text{raise exception}
raise \textit{exp} \hspace{1cm} \text{conditional}
\textit{if \exp}_1 \textit{then} \textit{exp}_2 \textit{else} \textit{exp}_3 \hspace{1cm} \text{iteration}
\textit{while \exp}_1 \textit{do} \textit{exp}_2 \hspace{1cm} \text{case analysis}
\textit{case \exp of \match} \hspace{1cm} \text{function}
\textit{fn match} \hspace{1cm} \text{function}

\textit{match} ::= \textit{mrule} \langle \mid \textit{match} \rangle
\textit{mrule} ::= \textit{pat} => \textit{exp}
\end{verbatim}

Figure 19: Grammar: Expressions and Matches
\[
dec ::= \text{val valbind} \quad \text{value declaration}
\quad \text{fun fvalbind} \quad \text{function declaration}
\quad \text{type typbind} \quad \text{type declaration}
\quad \text{datatype datbind (withtype typbind)} \quad \text{datatype declaration}
\quad \text{abstype datbind (withtype typbind)} \quad \text{abstype declaration}
\quad \text{with } \dec \text{ end} \quad \text{exception declaration}
\quad \text{local } \dec_1 \text{ in } \dec_2 \text{ end} \quad \text{local declaration}
\quad \text{open longstrid}_1 \cdots \text{longstrid}_n \quad \text{open declaration, } n \geq 1
\quad \dec_1 (\cdot) \dec_2 \quad \text{sequential declaration}
\quad \text{infix } (d) \text{ id}_1 \cdots \text{id}_n \quad \text{infix (L) directive, } n \geq 1
\quad \text{infixr } (d) \text{ id}_1 \cdots \text{id}_n \quad \text{infix (R) directive, } n \geq 1
\quad \text{nonfix } \text{id}_1 \cdots \text{id}_n \quad \text{nonfix directive, } n \geq 1
\]

\[
\text{valbind ::= pat = exp (and valbind)}
\quad \text{rec valbind}
\]

\[
\text{fvalbind ::= } \langle \text{op} \rangle \text{var atpat}_1 \cdots \text{atpat}_n \langle :ty \rangle = \text{exp}_1 \quad m, n \geq 1
\quad | \langle \text{op} \rangle \text{var atpat}_2 \cdots \text{atpat}_2n \langle :ty \rangle = \text{exp}_2
\quad | \quad \cdots \quad \cdots
\quad | \langle \text{op} \rangle \text{var atpat}_m \cdots \text{atpat}_mn \langle :ty \rangle = \text{exp}_m
\quad \langle \text{and fvalbind} \rangle
\]\n
\[
\text{typbind ::= tyvarseq tycon = ty (and typbind)}
\]

\[
\text{datbind ::= tyvarseq tycon = conbind (and datbind)}
\]

\[
\text{conbind ::= } \langle \text{op} \rangle \text{con } \langle \text{of ty} \rangle \langle | \text{conbind} \rangle
\]

\[
\text{exbind ::= } \langle \text{op} \rangle \text{excon } \langle \text{of ty} \rangle \langle \text{and exbind} \rangle
\quad \langle \text{op} \rangle \text{excon } = \langle \text{op} \rangle \longconexcon \langle \text{and exbind} \rangle
\]

Note: In the \text{fvalbind} form, if \text{var} has infix status then either \text{op} must be present, or \text{var} must be infixed. Thus, at the start of any clause, "\text{op var (atpat, atpat)} \cdots" may be written "(atpat var atpat) \cdots"; the parentheses may also be dropped if "\text{:ty}" or "\text{=}" follows immediately.

Figure 20: Grammar: Declarations and Bindings
\[\textit{atpat} ::= \_ \quad \text{wildcard}\]
\[\textit{scon} \quad \text{special constant}\]
\[\langle \text{op} \rangle \textit{var} \quad \text{variable}\]
\[\langle \text{op} \rangle \textit{longcon} \quad \text{constant}\]
\[\langle \text{op} \rangle \textit{longexcon} \quad \text{exception constant}\]
\[\{ \text{\textit{patrow} } \} \quad \text{record}\]
\[() \quad \text{0-tuple}\]
\[(\text{pat}_1, \ldots, \text{pat}_n) \quad \text{n-tuple, } n \geq 2\]
\[[\text{pat}_1, \ldots, \text{pat}_n] \quad \text{list, } n \geq 0\]
\[\langle \text{pat} \rangle \]

\[\textit{patrow} ::= \ldots \quad \text{wildcard}\]
\[\textit{lab} = \text{pat} \langle \text{, patrow} \rangle \quad \text{pattern row}\]
\[\textit{id} \langle \text{ty} \rangle \langle \text{as pat} \rangle \langle \text{, patrow} \rangle \quad \text{label as variable}\]

\[\textit{pat} ::= \text{atpat} \quad \text{atomic}\]
\[\langle \text{op} \rangle \textit{longcon atpat} \quad \text{value construction}\]
\[\langle \text{op} \rangle \textit{longexcon atpat} \quad \text{exception construction}\]
\[\text{pat}_1 \textit{con pat}_2 \quad \text{infixed value construction}\]
\[\text{pat}_1 \textit{excon pat}_2 \quad \text{infixed exception construction}\]
\[\text{pat} : \text{ty} \quad \text{typed}\]
\[\langle \text{op} \rangle \textit{var} \langle : \text{ty} \rangle \text{ as pat} \quad \text{layered}\]

Figure 21: Grammar: Patterns

\[\textit{ty} ::= \textit{tyvar} \quad \text{type variable}\]
\[\{ \langle \text{tyrow} \rangle \} \quad \text{record type expression}\]
\[\textit{tyseq longtycon} \quad \text{type construction}\]
\[\text{ty}_1 \ast \ldots \ast \text{ty}_n \quad \text{tuple type, } n \geq 2\]
\[\text{ty} \rightarrow \text{ty} \quad \text{function type expression}\]
\[() \]

\[\textit{tyrow} ::= \textit{lab} : \text{ty} \langle \text{, tyrow} \rangle \quad \text{type-expression row}\]

Figure 22: Grammar: Type expressions
Appendix: The Initial Static Basis

We shall indicate components of the initial basis by the subscript 0. The initial static basis is

\[ B_0 = (M_0, T_0, F_0, G_0, E_0) \]

where

- \( M_0 = \emptyset \)
- \( T_0 = \{ \text{bool}, \text{int}, \text{real}, \text{string}, \text{list}, \text{ref}, \text{exn}, \text{instream}, \text{outstream} \} \)
- \( F_0 = \{ \} \)
- \( G_0 = \{ \} \)
- \( E_0 = (SE_0, TE_0, VE_0, EE_0) \)

The members of \( T_0 \) are type names, not type constructors; for convenience we have used type-constructor identifiers to stand also for the type names which are bound to them in the initial static type environment \( TE_0 \). Of these type names, list and ref have arity 1, the rest have arity 0; all except exn, instream and outstream admit equality.

The components of \( E_0 \) are as follows:

- \( SE_0 = \{ \} \)
- \( VE_0 \) is shown in Figures 23 and 24. Note that \( \text{Dom} VE_0 \) contains those identifiers (true, false, nil, ::) which are basic value constructors, for reasons discussed in Section 4.3. \( VE_0 \) also includes \( EE_0 \), for the same reasons.
- \( TE_0 \) is shown in Figure 25. Note that the type structures in \( TE_0 \) contain the type schemes of all basic value constructors.
- \( \text{Dom} EE_0 = \text{BasExName} \), the set of basic exception names listed in Section 6.5. In each case the associated type is exn, except that \( EE_0(10) = \text{string} \to \text{exn} \).
<table>
<thead>
<tr>
<th>NONFIX</th>
<th>INFIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{var} \mapsto \sigma )</td>
<td>( \text{var} \mapsto \sigma )</td>
</tr>
<tr>
<td>map ( \mapsto \forall a\ '.b. ('a \rightarrow 'b) \rightarrow 'a \text{ list} \rightarrow 'b \text{ list} )</td>
<td>Precedence 7:</td>
</tr>
<tr>
<td>rev ( \mapsto \forall a. 'a \text{ list} \rightarrow 'a \text{ list} )</td>
<td>( / \mapsto \text{real} \times \text{real} \rightarrow \text{real} )</td>
</tr>
<tr>
<td>not ( \mapsto \text{bool} \rightarrow \text{bool} )</td>
<td>div ( \mapsto \text{int} \times \text{int} \rightarrow \text{int} )</td>
</tr>
<tr>
<td>( \sim \mapsto \text{num} \rightarrow \text{num} )</td>
<td>mod ( \mapsto \text{int} \times \text{int} \rightarrow \text{int} )</td>
</tr>
<tr>
<td>abs ( \mapsto \text{num} \rightarrow \text{num} )</td>
<td>( \ast \mapsto \text{num} \times \text{num} \rightarrow \text{num} )</td>
</tr>
<tr>
<td>floor ( \mapsto \text{real} \rightarrow \text{int} )</td>
<td>Precedence 6:</td>
</tr>
<tr>
<td>real ( \mapsto \text{int} \rightarrow \text{real} )</td>
<td>( + \mapsto \text{num} \times \text{num} \rightarrow \text{num} )</td>
</tr>
<tr>
<td>sqrt ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>( - \mapsto \text{num} \times \text{num} \rightarrow \text{num} )</td>
</tr>
<tr>
<td>sin ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>( \sim \mapsto \text{string} \times \text{string} \rightarrow \text{string} )</td>
</tr>
<tr>
<td>cos ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>Precedence 5:</td>
</tr>
<tr>
<td>arctan ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>( :: \mapsto \forall a.'a \times 'a \text{ list} \rightarrow 'a \text{ list} )</td>
</tr>
<tr>
<td>exp ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>( @ \mapsto \forall a.'a \text{ list} )</td>
</tr>
<tr>
<td>ln ( \mapsto \text{real} \rightarrow \text{real} )</td>
<td>( * 'a \text{ list} \rightarrow 'a \text{ list} )</td>
</tr>
<tr>
<td>size ( \mapsto \text{string} \rightarrow \text{int} )</td>
<td>Precedence 4:</td>
</tr>
<tr>
<td>chr ( \mapsto \text{int} \rightarrow \text{string} )</td>
<td>( = \mapsto \forall 'a. 'a \times 'a \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>ord ( \mapsto \text{string} \rightarrow \text{int} )</td>
<td>( &lt;&gt; \mapsto \forall 'a. 'a \times 'a \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>explode ( \mapsto \text{string} \rightarrow \text{string} )</td>
<td>( &lt; \mapsto \text{num} \times \text{num} \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>implode ( \mapsto \text{string list} \rightarrow \text{string} )</td>
<td>( &gt; \mapsto \text{num} \times \text{num} \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>( ! \mapsto \forall a. 'a \text{ ref} \rightarrow 'a )</td>
<td>( &lt;= \mapsto \text{num} \times \text{num} \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>ref ( \mapsto \forall 'a. '_a \rightarrow '_a \text{ ref} )</td>
<td>( &gt;= \mapsto \text{num} \times \text{num} \rightarrow \text{bool} )</td>
</tr>
<tr>
<td>true ( \mapsto \text{bool} )</td>
<td>Precedence 3:</td>
</tr>
<tr>
<td>false ( \mapsto \text{bool} )</td>
<td>( := \mapsto \forall a. 'a \text{ ref} \times 'a \rightarrow \text{unit} )</td>
</tr>
<tr>
<td>nil ( \mapsto \forall a. 'a \text{ list} )</td>
<td>( o \mapsto \forall a.'b.'c. ('b \rightarrow 'c) )</td>
</tr>
</tbody>
</table>

Notes:

- In type schemes we have taken the liberty of writing \( t y_1 \ast t y_2 \) in place of \( \{ 1 \mapsto t y_1, 2 \mapsto t y_2 \} \).

- An identifier with type involving \text{num} stands for two functions – one in which \text{num} is replaced by \text{int} in its type, and another in which \text{num} is replaced by \text{real} in its type. Sometimes an explicit type constraint will be needed if the surrounding text does not determine the type of a particular occurrence of + (for example). For this purpose, the surrounding text is no larger than the enclosing top-level declaration; an implementation may require that a smaller context determines the type.

Figure 23: Static VE_0 (except for Input/Output and EE_0)
<table>
<thead>
<tr>
<th>var</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>std_in</td>
<td>instream</td>
</tr>
<tr>
<td>open_in</td>
<td>string \rightarrow instream</td>
</tr>
<tr>
<td>input</td>
<td>instream \times int \rightarrow string</td>
</tr>
<tr>
<td>lookahead</td>
<td>instream \rightarrow string</td>
</tr>
<tr>
<td>close_in</td>
<td>instream \rightarrow unit</td>
</tr>
<tr>
<td>end_of_stream</td>
<td>instream \rightarrow bool</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>var</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>std_out</td>
<td>outstream</td>
</tr>
<tr>
<td>open_out</td>
<td>string \rightarrow outstream</td>
</tr>
<tr>
<td>output</td>
<td>outstream \times string \rightarrow unit</td>
</tr>
<tr>
<td>close_out</td>
<td>outstream \rightarrow unit</td>
</tr>
</tbody>
</table>

Figure 24: Static $VE_0$ (Input/Output)

<table>
<thead>
<tr>
<th>tycon</th>
<th>${ \theta, {con_1 \mapsto \sigma_1, \ldots, con_n \mapsto \sigma_n} }$ (n ≥ 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit</td>
<td>${ A().[], {} }$</td>
</tr>
<tr>
<td>bool</td>
<td>${ \text{bool}, { \text{true} \mapsto \text{bool}, \text{false} \mapsto \text{bool} } }$</td>
</tr>
<tr>
<td>int</td>
<td>${ \text{int}, {} }$</td>
</tr>
<tr>
<td>real</td>
<td>${ \text{real}, {} }$</td>
</tr>
<tr>
<td>string</td>
<td>${ \text{string}, {} }$</td>
</tr>
<tr>
<td>list</td>
<td>${ \text{list}, { \text{nil} \mapsto \forall \text{a}. \text{a list}, \text{::} \mapsto \forall \text{a}. \text{a} \times \text{a list} \rightarrow \text{a list} } }$</td>
</tr>
<tr>
<td>ref</td>
<td>${ \text{ref}, { \text{ref} \mapsto \forall \text{'_a}. \text{'_a} \rightarrow \text{'_a ref} } }$</td>
</tr>
<tr>
<td>exn</td>
<td>${ \text{exn}, {} }$</td>
</tr>
<tr>
<td>instream</td>
<td>${ \text{instream}, {} }$</td>
</tr>
<tr>
<td>outstream</td>
<td>${ \text{outstream}, {} }$</td>
</tr>
</tbody>
</table>

Figure 25: Static $TE_0$
D Appendix: The Initial Dynamic Basis

We shall indicate components of the initial basis by the subscript 0. The initial dynamic basis is

\[ B_0 = F_0, G_0, E_0 \]

where

- \( F_0 = \{ \} \)
- \( G_0 = \{ \} \)
- \( E_0 = E'_0 + E''_0 \)

\( E'_0 \) contains bindings of identifiers to the basic values BasVal and basic exception names BasExName; in fact \( E'_0 = SE'_0, VE'_0, EE'_0 \), where:

- \( SE'_0 = \{ \} \)
- \( VE'_0 = \{ id \mapsto id ; id \in \text{BasVal} \} \cup \{ : = \mapsto : = \} \)
- \( EE'_0 = \{ id \mapsto id ; id \in \text{BasExName} \} \)

Note that \( VE'_0 \) is the identity function on \( \text{BasVal} \); this is because we have chosen to denote these values by the names of variables to which they are initially bound. The semantics of these basic values (most of which are functions) lies principally in their behaviour under APPLY, which we describe below. On the other hand the semantics of \( : = \) is provided by a special semantic rule, rule 115. Similarly, \( EE'_0 \) is the identity function on \( \text{BasExName} \), the set of basic exception names, because we have also chosen these names to be just those exception constructors to which they are initially bound. These exceptions are raised by APPLY as described below.

\( E''_0 \) contains initial variable bindings which, unlike \( \text{BasVal} \), are definable in ML; it is the result of evaluating the following declaration in the basis \( F_0, G_0, E'_0 \). For convenience, we have also included all basic infix directives in this declaration.

\begin{verbatim}
infix 3 o
infix 4 = <> < > <= >=
infix 5 @
infixr 5 ::
infix 6 + - ^
infix 7 div mod / *

fun (F o G) x = F(G x)

fun nil @ M = M |
  (x::L) @ M = x::(L @ M)
\end{verbatim}
fun s ^ s' = implode((explode s) @ (explode s'))

fun map F nil = nil
  | map F (x::L) = (F x)::(map F L)

fun rev nil = nil
  | rev (x::L) = (rev L) @ [x]

fun not true = false
  | not false = true

fun ! (ref x) = x

We now describe the effect of APPLY upon each value \( b \in \text{BasVal} \). For special values, we shall normally use \( i, r, n, s \) to range over integers, reals, numbers (integer or real), strings respectively. We also take the liberty of abbreviating “\( \text{APPLY}(\text{abs}, r) \)” to “\( \text{abs}(r) \)”, “\( \text{APPLY}(\text{mod}, \{1 \mapsto i, 2 \mapsto d\}) \)” to “\( i \bmod d \)”, etc.

- \( "(n) \) returns the negation of \( n \), or the packet \([\text{Neg}]\) if the result is out of range.
- \( \text{abs}(n) \) returns the absolute value of \( n \), or the packet \([\text{Abs}]\) if the result is out of range.
- \( \text{floor}(r) \) returns the largest integer \( i \) not greater than \( r \); it returns the packet \([\text{Floor}]\) if \( i \) is out of range.
- \( \text{real}(i) \) returns the real value equal to \( i \).
- \( \text{sqrt}(r) \) returns the square root of \( r \), or the packet \([\text{Sqrt}]\) if \( r \) is negative.
- \( \text{sin}(r), \text{cos}(r) \) return the result of the appropriate trigonometric functions.
- \( \text{arctan}(r) \) returns the result of the appropriate trigonometric function in the range \( \pm \pi/2 \).
- \( \text{exp}(r), \ln(r) \) return respectively the exponential and the natural logarithm of \( r \), or an exception packet \([\text{Exp}]\) or \([\text{Ln}]\) if the result is out of range.
- \( \text{size}(s) \) returns the number of characters in \( s \).
- \( \text{chr}(i) \) returns the character numbered \( i \) (see Section 2.2) if \( i \) is in the interval \([0, 255]\), and the packet \([\text{Chr}]\) otherwise.
• \text{ord}(s)$ returns the number of the first character in $s$ (an integer in the interval $[0, 255]$, see Section 2.2), or the packet \{0\text{rd}\} if $s$ is empty.

• \text{explode}(s)$ returns the list of characters (as single-character strings) of which $s$ consists.

• \text{implode}(L)$ returns the string formed by concatenating all members of the list $L$ of strings.

• The arithmetic functions $\div, \times, +, -$ all return the results of the usual arithmetic operations, or exception packets respectively \{\text{Quot}\}, \{\text{Prod}\}, \{\text{Sum}\}, \{\text{Diff}\}$ if the result is undefined or out of range.

• $i \mod d, i \div d$ return integers $r, q$ (remainder, quotient) determined by the equation $d \times q + r = i$, where either $0 \leq r < d$ or $d < r \leq 0$. Thus the remainder has the same sign as the divisor $d$. The packet \{\text{Mod}\} or \{\text{Div}\}$ is returned if $d = 0$.

• The order relations $<, >, <=, >=$ return boolean values in accord with their usual meanings.

• $v_1 = v_2$ returns \text{true} or \text{false} according as the values $v_1$ and $v_2$ are, or are not, identical. The type discipline (in particular, the fact that function types do not admit equality) ensures that equality is only ever applied to special values, nullary constructors, addresses, and values built out of such by record formation and constructor application.

• $v_1 \leftrightarrow v_2$ returns the opposite boolean value to $v_1 = v_2$.

It remains to define the effect of APPLY upon basic values concerned with input/output; we therefore proceed to describe the ML input/output system.

Input/Output in ML uses the concept of a stream. A stream is a finite or infinite sequence of characters; if finite, it may or may not be terminated. (It may be convenient to think of a special end-of-stream character signifying termination, provided one realises that this “character” is never treated as data). Input streams – or instreams – are of type \text{instream} and will be denoted by \text{is}; output streams – or outstreams – are of type \text{outstream} and will be denoted by \text{os}. Both these types of stream are abstract, in the sense that streams may only be manipulated by the functions provided in BasVal.

Associated with an instream is a producer, normally an I/O device or file; similarly an outstream is associated with a consumer. After this association has been established – either initially or by the \text{open_in} or \text{open_out} function – the stream acts as a vehicle for character transmission from producer to program, or from program to consumer. The association can be broken by the \text{close_in} or \text{close_out} function. A closed stream permits no further character transmission; a closed instream is equivalent to one which is empty and terminated.
There are two streams in BasVal:

- **std_in**: an instream whose producer is the terminal.
- **std_out**: an outstream whose consumer is the terminal.

The other basic values concerned with Input/Output are all functional, and the effect of APPLY upon each of them given below. We take the liberty of abbreviating “APPLY(open_in, s)” to “open_in(s)” etc., and we shall use s and n to range over strings and integers respectively.

- **open_in(s)** returns a new instream is, whose producer is the external file named s. It returns exception packet
  
  
  \[
  ([\text{Io},"\text{Cannot open } s\text{"}])
  \]

  if file s does not exist or does not provide read access.

- **open_out(s)** returns a new outstream os, whose consumer is the external file named s. If file s is non-existent, it is taken to be initially empty.

- **input(is,n)** returns a string s containing the first n characters of is, also removing them from is. If only \( k < n \) characters are available on is, then
  
  - If is is terminated after these k characters, the returned string s contains them alone, and they are removed from is.
  
  - Otherwise no result is returned until the producer of is either supplies n characters or terminates the stream.

- **lookahead(is)** returns a single-character string s containing the next character of is, without removing it. If no character is available on is then
  
  - If is is closed, the empty string is returned.
  
  - Otherwise no result is returned until the producer of is either supplies a character or closes the stream.

- **close_in(is)** empties and terminates the instream is.

- **end_of_stream(is)** returns true if lookahead(is) returns the empty string, false otherwise; it detects the end of the instream is.

- **output(os,s)** writes the characters of s to the outstream os, unless os is closed, in which case it returns the exception packet
  
  \[
  ([\text{Io},"\text{Output stream is closed"}])
  \]

- **close_out(os)** terminates the outstream os.
E Appendix: The Development of ML

This Appendix records the main stages in the development of ML, and the people principally involved. The main emphasis is upon the design of the language; there is also a section devoted to implementation. On the other hand, no attempt is made to record work on implementation environments, or on applications of the language.

Origins

ML and its semantic description have evolved over a period of about fourteen years. It is a fusion of many ideas from many people; in this appendix we try to record and to acknowledge the important precursors of its ideas, the important influences upon it, and the important contributions to its design, implementation and semantic description.

ML, which stands for meta language, was conceived as a medium for finding and performing proofs in a formal logical system. This application was the focus of the initial design effort, by Robin Milner in collaboration first with Malcolm Newey and Lockwood Morris, then with Michael Gordon and Christopher Wadsworth [11]. The intended application to proof affected the design considerably. Higher order functions in full generality seemed necessary for programming proof tactics and strategies, and also a robust type system (see below). At the same time, imperative features were important for practical reasons; no-one had experience of large useful programs written in a pure functional style. In particular, an exception-raising mechanism was highly desirable for the natural presentation of tactics.

The full definition of this first version of ML was included in a book [12] which describes LCF, the proof system which ML was designed to support. The details of how the proof application exerted an influence on design is reported by Milner [24]. Other early influences were the applicative languages already in use in Artificial Intelligence, principally LISP [21], ISWIM [19] and POP2 [5].

Polymorphic types

The polymorphic type discipline and the associated type-assignment algorithm were prompted by the need for security; it is vital to know that when a program produces an object which it claims to be a theorem, then it is indeed a theorem. A type discipline provides the security, but a polymorphic discipline also permits considerable flexibility.

The key ideas of the type discipline were evolved in combinatory logic by Haskell Curry and Roger Hindley, who arrived at different but equivalent algorithms for computing principal type schemes. Curry's [7] algorithm was by equation-solving; Hindley [14] used the unification algorithm of Alan Robinson
[29] and also presented the precursor of our type inference system. James Morris [26] independently gave an equation-solving algorithm very similar to Curry's. The idea of an algorithm for finding principal type schemes is very natural and may well have been known earlier. I am grateful to Roger Hindley for pointing out that Carew Meredith's inference rule for propositional logic called Condensed Detachment, defined in the early 1950s, clearly suggests that he knew such an algorithm [22].

Milner [23], during the design of ML, rediscovered principal types and their calculation by unification, for a language (slightly richer than combinatory logic) containing local declarations. He and Damas [9] presented the ML type inference systems following Hindley's style. Damas [8], using ideas from Michael Gordon, also devised the first mathematical treatment of polymorphism in the presence of references and assignment. Tofte [32] produced a different scheme, which has been adopted in the language.

Refinement of the Core Language

Two movements led to the re-design of ML. One was the work of Rod Burstall and his group on specifications, crystallised in the specification language CLEAR [4] and in the functional programming language HOPE [3]; the latter was for expressing executable specifications. The outcome of this work which is relevant here was twofold. First, there were elegant programming features in HOPE, particularly pattern matching and clausal function definitions; second, there were ideas on modular construction of specifications, using signatures in the interfaces. A smaller but significant movement was by Luca Cardelli, who extended the data-type repertoire in ML by adding named records and variant types.

In 1983, Milner (prompted by Bernard Sumer) wrote the first draft of a standard form of ML attempting to unite these ideas; over the next three years it evolved into the Standard ML Core Language. Notable here was the harmony found among polymorphism, HOPE patterns and Cardelli records, and the nice generalisations of ML exceptions due to ideas from Alan Mycroft, Brian Monahan and Don Sannella. A simple stream-based I/O mechanism was developed from ideas of Cardelli by Milner and Harper. The Standard ML Core Language is described in detail in a composite report [15] which also contains a description of the I/O mechanism and MacQueen's proposal for program modules (see later for discussion of this). Since then only few changes to the Core Language have occurred. Milner proposed equality types, and these were added, together with a few minor adjustments [25]. The latest and final development has been in the exception mechanism, by MacQueen using an idea from Burstall [1]; it unites the ideas of exception and data type construction.
Modules

Besides contributory ideas to the Core Language, HOPE [3] contained a simple notion of program module. The most important and original feature of ML Modules, however, stems from the work on parameterised specifications in CLEAR [4]. MacQueen, who was a member of Burstall’s group at the time, designed [20] a new parametric module feature for HOPE inspired by the CLEAR work. He later extended the parameterisation ideas by a novel method of specifying sharing of components among the structure parameters of a functor, and produced a draft design which accommodated features already present in ML – in particular the polymorphic type system. This design was discussed in detail at Edinburgh, leading to MacQueen’s first report on Modules [15].

Thereafter, the design came under close scrutiny through a draft operational static semantics and prototype implementation of it by Harper, through Kevin Mitchell’s implementation of the evaluation, through a denotational semantics written by Don Sannella, and then through further work on operational semantics by Milner and Tofte. (More is said about this in the later section on Semantics.) In all of this work the central ideas withstood scrutiny, while it also became clear that there were gaps in the design and ambiguities in interpretation. (An example of a gap was the inability to specify sharing between a functor argument structure and its result structure; an example of an ambiguity was the question of whether sharing exists in a structure over and above what is specified in the signature expression which accompanies its declaration.)

Much discussion ensued; it was possible for a wider group to comment on Modules through using Harper’s prototype implementation, while Harper, Milner and Tofte gained understanding during development of this semantics. In parallel, Sannella and Tarlecki explored the implications of Modules for the methodology of program development [30]. Tofte, in his thesis [31], proved several technical properties of Modules in a skeletal language, which generated considerable confidence in this design. A key point in this development was the proof of the existence of principal signatures, and, in the careful distinction between the notion of enrichment of structures, which allows more polymorphism and more components, and realisation which allows more sharing.

At a meeting in Edinburgh in 1987 a choice of two designs was presented, hinging upon whether or not a functor application should coerce its actual argument to its argument signature. The meeting chose coercion, and thereafter the production of Section 5 of this report – the Static Semantics of Modules – was a matter of detailed care. That section is undoubtedly the most original and demanding part of this semantics, just as the ideas of MacQueen upon which it is based are the most far-reaching extension to the original design of ML.
Implementation

The first implementation of ML was by Malcolm Newey, Lockwood Morris and Robin Milner in 1974, for the DEC10. Later Mike Gordon and Chris Wadsworth joined; their work was mainly in specialising ML towards machine-assisted reasoning. Around 1980 Luca Cardelli implemented a version on VAX; his work was later extended by Alan Mycroft, Kevin Mitchell and John Scott. This version contained one or two new data-type features, and was based upon the Functional Abstract Machine (FAM), a virtual machine which has been a considerable stimulus to later implementation. By providing a reasonably efficient implementation, this work enabled the language to be taught to students; this, in turn, prompted the idea that it could become a useful general purpose language.

In Gothenburg, an implementation was developed by Lennart Augustsson and Thomas Johnsson in 1982, using lazy evaluation rather than call-by-value; the result was called Lazy ML and is reported in [2]. This work is part of continuing research in many places on implementation of lazy evaluation in pure functional languages. But for ML, which includes exceptions and assignment, the emphasis has been mainly upon strict evaluation (call-by-value).

In Cambridge, in the early 1980s, Larry Paulson made considerable improvements to the Edinburgh ML compiler, as part of his wider programme of improving Edinburgh LCF to become Cambridge LCF [27]. This system has supported larger proofs than the Edinburgh system, and with greater convenience; in particular, the compiled ML code ran four to five times faster.

Around the same time Gérard Huet at INRIA (Versailles) adapted ML to Maclisp on Multics, again for use in machine-assisted proof. There was close collaboration between INRIA and Cambridge in this period. ML has undergone a separate development in the group at INRIA, arriving at a language and implementation known as CAML [6]; this is close to the core language of Standard ML, but does not include the Modules.

The first implementation of the Standard ML core language was by Mitchell, Mycroft and John Scott of Edinburgh, around 1984, and this was shortly followed by an implementation by David Matthews at Cambridge, carried out in his language Poly.

The prototype implementation of Modules, before that part of the language settled down, was done in 1985-6; Mitchell dealt with evaluation, while Harper tackled the elaboration (or ‘signature checking’) which raised problems of a kind not previously encountered. The Edinburgh implementation continues to play the role of a test-bed for language development.

Meanwhile Matthews’ Cambridge implementation also advanced to embrace Modules. This implementation has supported applications of considerable size, both for machine-assisted proof and for hardware design.

In 1986, as the Modules definition was settling down, David MacQueen began an implementation at Bell AT&T Laboratories, joined later by Andrew Appel
and Trevor Jim who are particularly interested in compilation into high quality machine code.

The Bell and Cambridge implementations, the former led by MacQueen and Appel, the latter by Matthews, are currently the most complete and highly engineered. Other currently active implementations are by Michael Hedlund at the Rutherford-Appleton Laboratory, by Robert Duncan, Simon Nichols and Aaron Sloman at the University of Sussex (POPLOG) and by Malcolm Newey and his group at the Australian National University.

Semantics

The description of the first version of ML [12] was informal, and in an operational style; around the same time a denotational semantics was written, but never published, by Mike Gordon and Robin Milner. Meanwhile structured operational semantics, presented as an inference system, was gaining credence as a tractable medium. This originates with the reduction rules of λ-calculus, but was developed more widely through the work of Plotkin [28], and also by Milner. This was at first only used for dynamic semantics, but later the benefit of using inference systems for both static and dynamic semantics became apparent. This advantage was realised when Gilles Kahn and his group at INRIA were able to execute early versions of both forms of semantics for the ML Core Language using their Typol system [10]. The static and dynamic semantics of the Core reached a final form mostly through work by Mads Tofte and Robin Milner.

The modules of ML presented little difficulty as far as dynamic semantics is concerned, but the static semantics of Modules was a concerted effort by several people. MacQueen's original informal description [15] was the starting point; Sannella wrote a denotational semantics for several versions, which showed that several issues had not been settled by the informal description. Robert Harper, while writing the first implementation of Modules, made the first draft of the static semantics. Harper's version made clear the importance of structure names; work by Milner and Tofte introduced further ideas including realisation; thereafter a concerted effort by all three led to several suggestions for modification of the language, and a small range of alternative interpretations; these were assessed in discussion with MacQueen, and more widely with the principal users of the language, and an agreed form was reached.

There is no doubt that the interaction between design and semantic description of Modules has been one of the most striking phases in the entire language development, leading (in the opinion of those involved) to a high degree of confidence both in the language and in the semantic method.
Literature

The present document is the definition of Standard ML; further versions of it will be produced as the language develops (but the intention is to minimise the number of versions). An informal definition, consistent with Version 2 of this document as far as the Core Language is concerned, is provided by [15], as modified by [25] and [1]. An elementary textbook covering the Core language has been recently published, written by Åke Wikström [33]. Robert Harper [13] has written a shorter introduction which also includes material on Modules.

Further acknowledgments

Apart from the people mentioned above we also acknowledge the following, all of whom have contributed in some way to the evolution of ML: Guy Cousineau, Simon Finn, Jim Hook, Gerard Huet, Gilles Kahn, Brian Monahan, Peter Mosses, Alan Mycroft, David Park, David Rydeheard, David Schmidt, Stefan Sokolowski, Bernard Sufrin, Philip Wadler.
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