# Exercises

**Exercise 1**

Consider the subtour elimination constraint defined in proposition 1 (section 4.2) in the paper. For the set $S = \{i, j, k\}$ (that is, $h = 3$) it has the following form (see figure 3 in the paper):

$$x_{ij} + x_{jk} + x_{ki} + 2x_{ji} + x_{n+j,i} + x_{n+k,i} \leq 2$$

Is it possible to add any nontrivial edges to the left hand side without increasing the right hand side such that the inequality still is valid? By nontrivial I mean edges that would not already be eliminated by the preprocessing described in section 5.1. An example of an trivial edge is the edge from the start terminal to a delivery.

You should only consider the case where $S$ contains three nodes. It is not neceassary to generalize the result.

The next three exercises ask you to find violated inequalities in fractional solutions. The fractional solutions are shown in figures 1 and 2. The rectangles in the figures represent the start and end terminal. The ellipses represent pickups and the triangles represent deliveries. The number in the parenthesis is the node number (as defined in the paper). The last number in the pickup and delivery nodes is the amount picked up (indicated with a "+") or amount delivered (indicated with a "-"). The weight of an edge $(i, j)$ represents the value of the variable $x_{ij}$ in the fractional solution (or alternatively: the value of $\sum_{k \in K} x_{ij}^k$ if you think in terms of $x_{ij}^k$ variables).

Hints to exercise 2-4 can be found at the CAOS website: I recommend that you try to solve the exercise without using the hints first, though. That is most fun!

**Exercise 2**

Find a violated subtour elimination constraint (see section 4.2 in the paper) in the fractional solution shown in figure 1.

**Exercise 3**

Find a violated capacity constraint (see section 4.3 in the paper) in the fractional solution shown in figure 2. The capacity of the vehicle is three units.

**Exercise 4**

Find a violated generalized order constraint (see section 4.5 in the paper) in the fractional solution shown in figure 1.

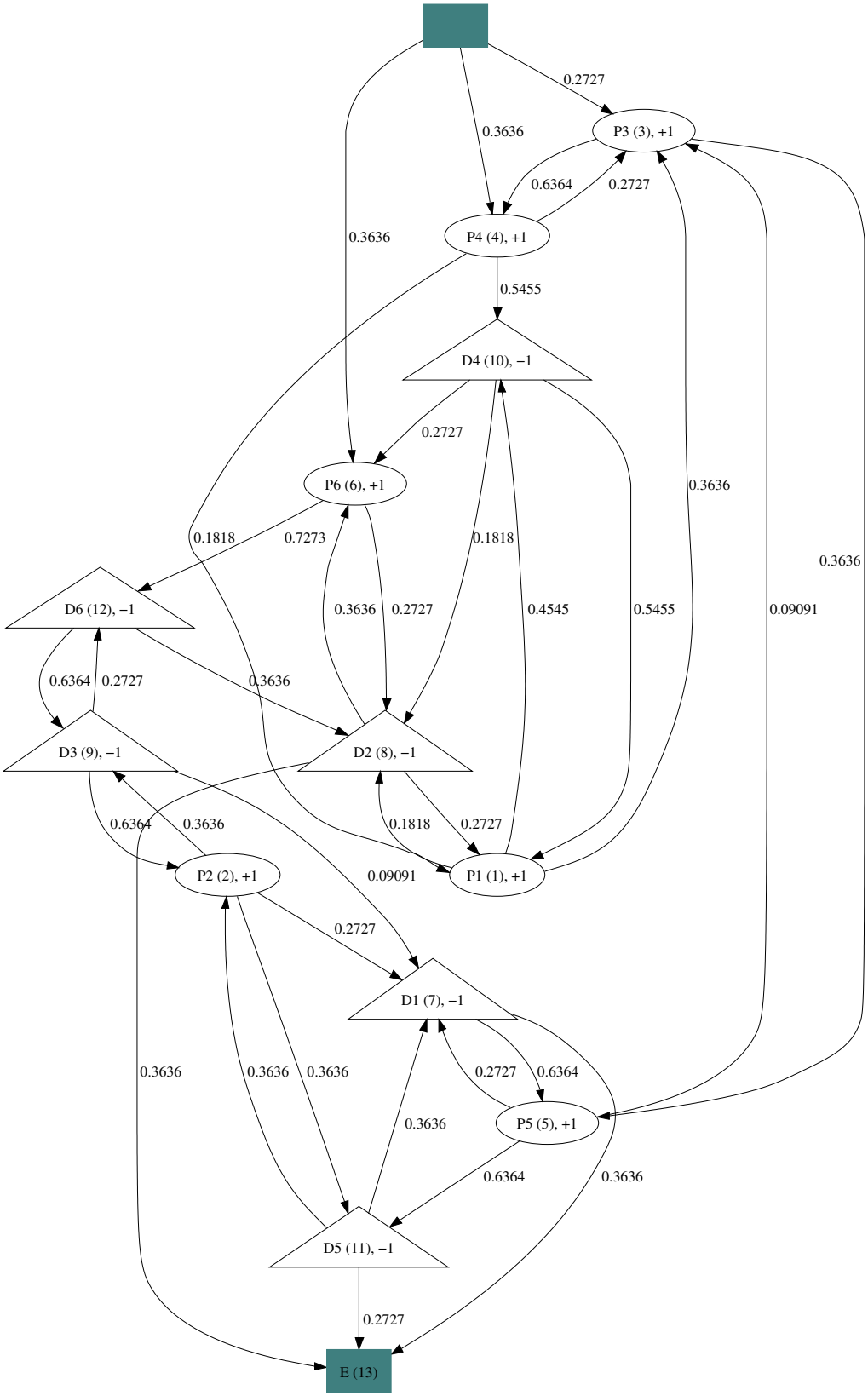


Figure 1: Fractional solution to a DARP problem with 6 requests.

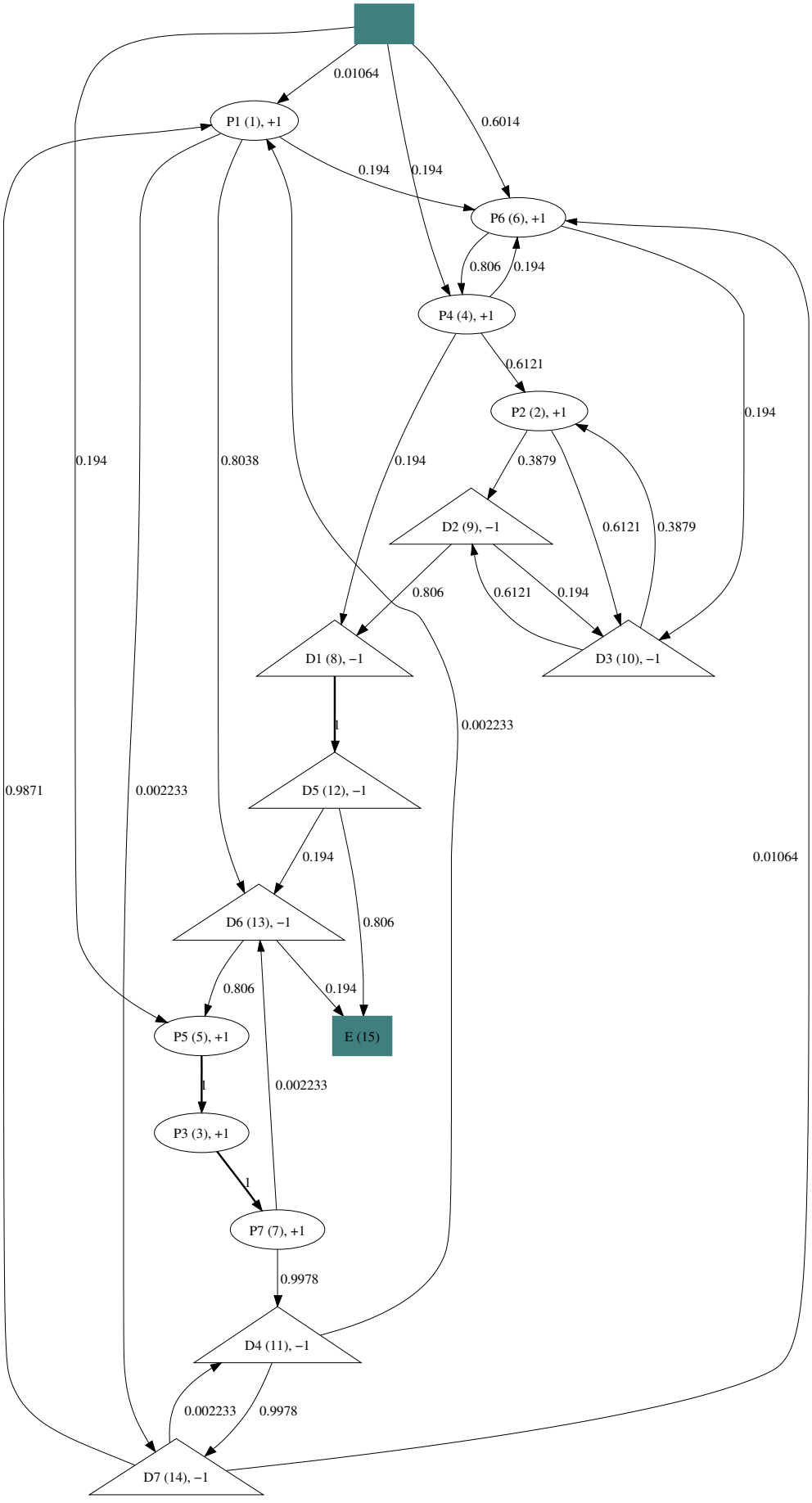


Figure 2: Fractional solution to a DARP problem with 7 requests.