The Capacitated Zero-Shift Problem

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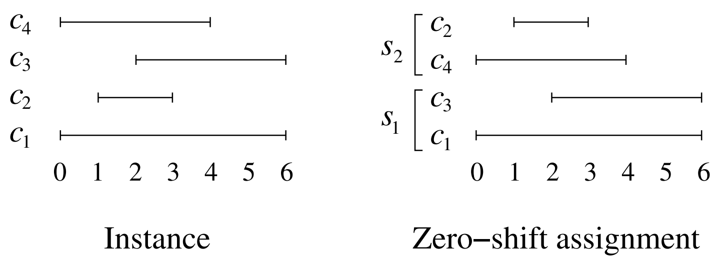
There are more than 18 million containers in the world today. Containers are stowed in stacks that only can be accessed from the top. For that reason, a major challenge is to minimize the number of containers that must be removed (or *shifted*) in order to reach containers that need to be retrieved.

The capacitated zero-shift problem (CZP) is a decision problem that asks whether a set of containers that must be stored and retrieved at discrete time points can be stowed without shifting in a fixed number of stacks with limited capacity. Formally, an instance of the CZP is a tuple $\langle C, in, out, S, m \rangle$, where C is a finite set of containers, $in(c) \in \mathbb{N}$ ($out(c) \in \mathbb{N}$) is the time point that container c must be stowed in (retrieved from) one of the stacks, S is a finite set of stacks, and $m \in \mathbb{N}$ is the maximum number of containers that each stack can hold at any time.

The question is whether the containers can be assigned to the stacks such that no shifting is required to retrieve them. Formally, the question is whether there exists an assignment $A : C \rightarrow S$ that is within the stack capacity (i.e., $\forall t \in \mathbb{N}, s \in S \,.\, |\{ c \mid A(c) = s, in(c) \leq t < out(c)\}| \leq m$) and requires no shifting (i.e., $\nexists v, w \,.\, A(v) = A(w) \wedge in(v) < in(w) < out(v) \wedge in(w) < out(v) < out(w)$).

As an example, consider a CZP with $C = \{c_1, c_2, c_3, c_4\}$, $in(c_1) = 0$, $in(c_2) = 1$, $in(c_3) = 2$, $in(c_4) = 0$, $out(c_1) = 6$, $out(c_2) = 3$, $out(c_3) = 6$, $out(c_4) = 4$, $S = \{s_1, s_2\}$, and $m = 2$. As depicted in the figure below, the answer to this CZP is "yes", because the shown assignment is within the stack capacity and requires no shifting.



The uncapacitated version of the CZP where $m = \infty$ has been shown to be NP-complete for $|S| \geq 4$ by a reduction from coloring of overlap graphs [2] and is known to be polynomial for $|S| < 4$ [1, 4]. For $m = 1$, the CZP is solvable in polynomial time using a minimal cost flow formulation for interval scheduling on identical machines [3]. But even for $m = 2$, neither a polynomial time algorithm nor an NP-completeness proof is known.

In other words, while the complexity of many problems related to storing data items on electronic devices is well known, we know surprisingly little about the complexity of storing physical items in the common way of container stacks.

References

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[4] W. Unger. The complexity of coloring circle graphs. In *Proceedings of STACS 92*, volume 577 of *Lecture Notes in Computer Science*, pages 389–400. Springer, 1992.