Programs as data
Parsing cont'd; first-order functional language, type checking

Peter Sestoft

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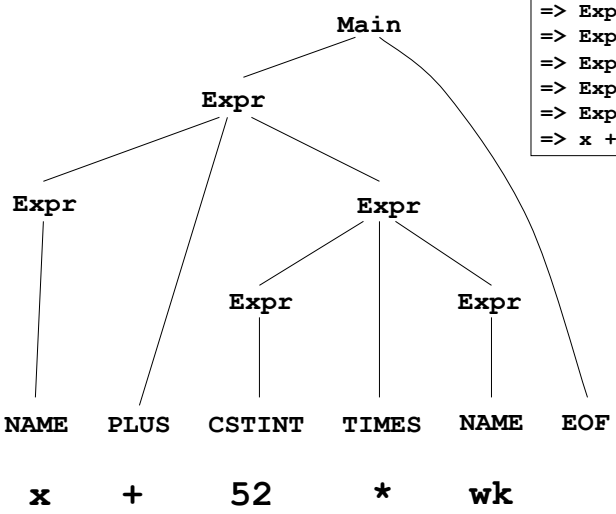
Plan for today

- Parsing:
  - LR versus LL
  - How does an LR parser work
  - Hand-writing an LL parser
- A first-order functional language
  - Lexer and parser specifications
  - Interpretation: function closures
- Explicit types
  - a type checking function
  - type rules
- Static versus dynamic types

LR versus LL parsing

- **LR**: Read input from **L**eft to right, make derivations from **R**ightmost nonterminal
  - Bottom-up parsing
  - Difficult to hand-write parsers, but excellent parser generator tools – e.g. fsyacc – exist
  - No grammar transformations required
- **LL**: Read input from **L**eft to right, make derivations from **L**eftmost nonterminal
  - Top-down parsing
  - Fairly easy to hand-write a parser
  - *But* requires grammar transformations, to encode associativity and precedence

An LR derivation from last week




```
Main
=> Expr EOF                    A
=> Expr + Expr EOF             H
=> Expr + Expr * Expr EOF      G
=> Expr + Expr * wk EOF        B
=> Expr + 52 * wk EOF          C
=> x + 52 * wk EOF             B
```

Derivation tree

The (fsyacc, LR) parser automaton

- A parser is an automaton with a stack

```
state 19:
    items:
      Expr -> Expr . 'TIMES' Expr
      Expr -> Expr . 'PLUS' Expr
      Expr -> Expr 'PLUS' Expr .
      Expr -> Expr . 'MINUS' Expr
    actions:
      action 'EOF':    reduce Expr --> Expr 'PLUS' Expr
      action 'LPAR':   reduce Expr --> Expr 'PLUS' Expr
      action 'RPAR':   reduce Expr --> Expr 'PLUS' Expr
      action 'END':    reduce Expr --> Expr 'PLUS' Expr
      action 'IN':     reduce Expr --> Expr 'PLUS' Expr
      action 'LET':    reduce Expr --> Expr 'PLUS' Expr
      action 'PLUS':   reduce Expr --> Expr 'PLUS' Expr
      action 'MINUS':  reduce Expr --> Expr 'PLUS' Expr
      action 'TIMES':  shift 21
      action 'EQ':     reduce Expr --> Expr 'PLUS' Expr
      action 'NAME':   reduce Expr --> Expr 'PLUS' Expr
      action 'CSTINT': reduce Expr --> Expr 'PLUS' Expr
      action 'error':  reduce Expr --> Expr 'PLUS' Expr
      action '#':      reduce Expr --> Expr 'PLUS' Expr
      action '$$':     reduce Expr --> Expr 'PLUS' Expr
    immediate action: <none>
    gotos:
```

One state

LR item set

State's action table

File **ExprPar.fsyacc.output** from **fsyacc –v ExprPar.fsy**

Parser stack snapshots, example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2
EOF	#0 Expr #2	shift 3
	#0 Expr #2 EOF #3	reduce by A
	#0 Main	goto #1
	#0 Main #1	accept

- Order of reduce actions is reverse LR derivation!

Parser state and actions

- Parser state = parser stack, containing:
  - Parser state numbers: #n
  - Grammar symbols: terminals and nonterminals

Parser actions:

- *Shift*: read a symbol from input onto the stack, and goto new state
- *Reduce*: take grammar rule rhs symbols off the stack and replace them by its lhs nonterminal, and evaluate a semantic action
- *Goto*: go to a new parser state (after reduce)

Shift/reduce conflicts

- Sometimes the parser generator does not know whether to shift or to reduce
- Especially if the grammar is ambiguous
- Then warnings are issued by `fsyacc`

- To resolve shift/reduce conflicts, change the parser specification
- To understand how, study the parser automaton in ExprPar.fsyacc.output

LL parsing, recursive descent

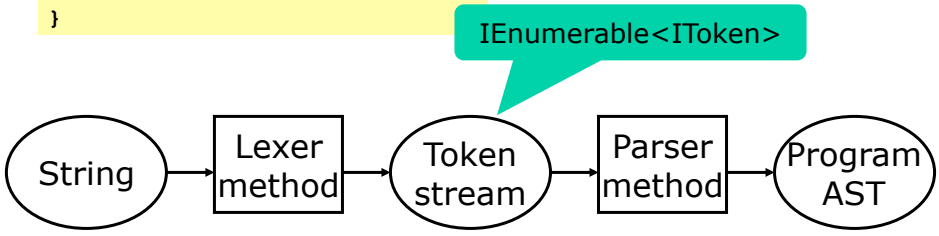
- Example: Scheme terms, ”S-expressions”
  - *symbols* such as `foo`, `bar`, `b52`, `+`, `*`
  - *numbers* such as `117`, `-4`
  - nested *lists* such as `(foo (+ n 1))`
- Grammar:

```
sexp ::= symbol
       | number
       | ( sexp* )
```


Hand-written lexer and parser in C#

- A token is an object implementing IToken

```
interface IToken { }
class Lpar : IToken { ... }
class Rpar : IToken { ... }
class Symbol : IToken {
  public readonly String name;
  ...
}
class NumberCst : IToken {
  public readonly int val;
  ...
}
```



Handwritten lexer (tokenizer)

```
public static IEnumerator<IToken> Tokenize(TextReader rd) {
  for (;;) {
    int raw = rd.Read();
    char ch = (char)raw;
    if (raw == -1)
      yield break;
    else if (Char.IsWhiteSpace(ch))
      { }
    else if (Char.IsDigit(ch))
      yield return new NumberCst(ScanNumber(ch, rd));
    else switch (ch) {
      case '(':
        yield return Lpar.LPAR; break;
      case ')':
        yield return Rpar.RPAR; break;
      case '-': // negative number, or symbol
        ...
      default:
        yield return ScanSymbol(ch, rd);
        break;
} } }
```

Helper method to make a number token

Parsing S-expressions top-down

```
sexp ::= symbol
       | number
       | ( sexp* )
```

- To parse S-expression:
- If next token is Symbol, then success
- If next token is NumberCst, then success
- If next token is Lpar, then
  - read that token
  - while next token is not Rpar
    - parse an S-expression
- If next token is anything else, then error

Handwritten recursive descent parser

```
public static void ParseSexp(IEnumerator<IToken> ts) {
  if (ts.Current is Symbol) {
    Console.WriteLine("Parsed symbol " + ts.Current);
  } else if (ts.Current is NumberCst) {
    Console.WriteLine("Parsed number " + ts.Current);
  } else if (ts.Current is Lpar) {
    Console.WriteLine("Started parsing list");
    Advance(ts);
    while (!(ts.Current is Rpar)) {
      ParseSexp(ts);
      Advance(ts);
    }
    Console.WriteLine("Ended parsing list");
  } else
    throw new ArgumentException("Parse error");
}
```

```
private static void Advance(IEnumerator<IToken> ts) {
  if (!ts.MoveNext())
    throw new ArgumentException("Unxpected eof");
}
```

Grammar classes (Chomsky hierarchy, 1956)

- Type 3: Regular grammars; same expressiveness as regular expressions

  $A \rightarrow cB$ $A \rightarrow B$ $A \rightarrow c$ $A \rightarrow \varepsilon$
- Type 2: Context-free grammars (CFG)

  $A \rightarrow cBd$
- Type 1: Context-sensitive grammars, non-abbreviating rules

  $aAb \rightarrow acAdb$
- Type 0: Unrestricted grammars; same as term rewrite systems

  $0Ay \rightarrow 0$

Micro-ML: A small functional language

- First-order: A value cannot be a function
- Dynamically typed, so this is OK:

```
if true then 1+2 else 1+false
```

- Eager, or call-by-value: In a call `f(e)` the argument `e` is evaluated before `f` is called
- Example Micro-ML programs (an F# subset):

```
5+7

let f x = x + 7 in f 2 end

let fac x = if x=0 then 1 else x * fac(x - 1)
in fac 10 end
```


Abstract syntax of Micro-ML

```
type expr =
  | CstI of int
  | CstB of bool
  | Var of string
  | Let of string * expr * expr
  | Prim of string * expr * expr
  | If of expr * expr * expr
  | Letfun of string * string * expr * expr
  | Call of expr * expr
```

```
let f x = x + 7 in f 2 end
```

(f, x, fBody, letBody)

```
Letfun ("f", "x", Prim ("+",Var "x",CstI 7),
        Call (Var "f",CstI 2))
```

Runtime values, function closures

- Run-time values: integers and functions

```
type value =
  | Int of int
  | Closure of string * string * expr * value env
```

- *Closure*: a package of a function's body and its declaration environment
- A name should refer to a *statically* enclosing binding:

```
let y = 11
in let f x = x + y
   in let y = 22 in f 3 end
   end
end
```

Should always have value 11

Evaluate as 3 + y

(f, x, x+y, [(y,11)])

Interpretation of Micro-ML

- Constants, variables, primitives, let, if: as for expressions
- Letfun: Create function closure and bind f to it
- Function call f(e):
  - Look up f, it must be a closure
  - Evaluate e
  - Create environment and evaluate the function's body

```
let rec eval (e : expr) (env : value env) : int =
    match e with
    | ...
    | Letfun(f, x, fBody, letBody) ->
      let bodyEnv = (f, Closure(f, x, fBody, env)) :: env
      in eval letBody bodyEnv
    | Call(Var f, eArg) ->
      let fClosure = lookup env f
      in match fClosure with
         | Closure (f, x, fBody, fDeclEnv) ->
           let xVal = Int(eval eArg env)
           let fBodyEnv = (x, xVal) :: (f, fClosure) :: fDeclEnv
           in eval fBody fBodyEnv
         | _ -> failwith "eval Call: not a function"
```

Evaluate fBody in declaration environment

Dynamic scope (instead of static)

- With static scope, a variable refers to the lexically, or statically, most recent binding
- With **dynamic scope**, a variable refers to the dynamically most recent binding:

```
let y = 11
in let f x = x + y
   in let y = 22 in f 3 end
   end
end
```

Evaluate as
3 + y

A dynamic scope variant of Micro-ML

- Very minimal change in interpreter:

```
let rec eval (e : expr) (env : value env) : int =
    ...
    | Call(Var f, eArg) ->
      let fClosure = lookup env f
      in match fClosure with
         | Closure (f, x, fBody, fDeclEnv) ->
           let xVal = Int(eval eArg env)
           let fBodyEnv = (x, xVal) :: (f, fClosure) :: env
           in eval fBody fBodyEnv
```

Evaluate fBody in call environment

- fDeclEnv is ignored; function is just (f, x, fBody)
- Good and bad:
  - simple to implement (no closures needed)
  - makes type checking difficult
  - makes efficient implementation difficult
- Used in macro languages, and Lisp, Perl, Clojure

Lexer and parser for Micro-ML

- Lexer:
  - Nested comments, as in F#, Standard ML

```
1 + (* 33 (* was 44 *) *) 22
```

- Parser:
  - To parse applications e1 e2 e3 correctly, distinguish atomic expressions from others

- Problem: f(x-1) parses as f(x(-1))
- Solution:
  - FunLex.fsl: make **`CSTINT`** just **`[0-9]+`** without sign
  - FunPar.fsy: add rule **`Expr := MINUS Expr`**

An explicitly typed fun. language

```
let f (x : int) : int = x+1
in f 12 end
```

```
type typ =
  | TypI
  | TypB
  | TypF of typ * typ
```

```
Letfun("f", "x", TypI,
       Prim("+", Var "x", CstI 1), TypI,
       Call(Var "f", CstI 12));;
```

(TypF(TypI, TypI)

(f, x, xTyp, fBody, rTyp, letBody

```
type tyexpr =
  | CstI of int
  | CstB of bool
  | Var of string
  | Let of string * tyexpr * tyexpr
  | Prim of string * tyexpr * tyexpr
  | If of tyexpr * tyexpr * tyexpr
  | Letfun of string * string * typ * tyexpr * typ * tyexpr
  | Call of tyexpr * tyexpr
```

Type checking by recursive function

- Using a type environment [("x", TypI)]:

```
let rec typ (e : tyexpr) (env : typ env) : typ =
    match e with
    | CstI i -> TypI
    | CstB b -> TypB
    | Var x  -> lookup env x
    | Prim(ope, e1, e2) ->
      let t1 = typ e1 env
      let t2 = typ e2 env
      in match (ope, t1, t2) with
          | ("*",  TypI, TypI) -> TypI
          | ("+",  TypI, TypI) -> TypI
          | ("-",  TypI, TypI) -> TypI
          | ("=",  TypI, TypI) -> TypB
          | ("<",  TypI, TypI) -> TypB
          | ("&&", TypB, TypB) -> TypB
          | _    -> failwith "unknown primitive, or type error"
    | ...
```


Type checking, part 2

- Checking `let x=eRhs in letBody end`
- Checking `if e1 then e2 else e3`

```
let rec typ (e : tyexpr) (env : typ env) : typ =
    match e with
    | Let(x, eRhs, letBody) ->
      let xTyp = typ eRhs env
      let letBodyEnv = (x, xTyp) :: env
      in typ letBody letBodyEnv
    | If(e1, e2, e3) ->
      match typ e1 env with
       | TypB -> let t2 = typ e2 env
                 let t3 = typ e3 env
                 in if t2 = t3 then t2
                    else failwith "If: branch types differ"
       | _    -> failwith "If: condition not boolean"
    | ...
```

Type checking, part 3

- Checking **`let f x=eBody in letBody end`**
- Checking **`f eArg`**

```
let rec typ (e : tyexpr) (env : typ env) : typ =
    match e with
    | ...
    | Letfun(f, x, xTyp, fBody, rTyp, letBody) ->
      let fTyp = TypF(xTyp, rTyp)
      let fBodyEnv = (x, xTyp) :: (f, fTyp) :: env
      let letBodyEnv = (f, fTyp) :: env
      in if typ fBody fBodyEnv = rTyp then typ letBody letBodyEnv
         else failwith "Letfun: wrong return type in function"
    | Call(Var f, eArg) ->
      match lookup env f with
       | TypF(xTyp, rTyp) ->
         if typ eArg env = xTyp then rTyp
         else failwith "Call: wrong argument type"
       | _ -> failwith "Call: unknown function"
    | Call(_, eArg) -> failwith "Call: illegal function in call"
```

Type checking versus evaluation

- The type checker **`typ`** and the interpreter **`eval`** have similar structure
- Type checking can be thought of as *abstract interpretation* of the program
- We calculate "TypI + TypI gives TypI" instead of "Int 3 + Int 5 gives Int 8"
- One major difference:
  - Type checking a function call f(e) does not require type checking the function's body again
  - Interpreting a function call f(e) does require interpreting the function's body
- Type checking always terminates

Type checking by logical rules

$$\rho \vdash i : \texttt{int}$$

$$\rho \vdash b : \texttt{bool}$$

$$\frac{\rho(x) = t}{\rho \vdash x : t}$$

$$\frac{\rho \vdash e_1 : \texttt{int} \qquad \rho \vdash e_2 : \texttt{int}}{\rho \vdash e_1 + e_2 : \texttt{int}}$$

$$\frac{\rho \vdash e_1 : \texttt{int} \qquad \rho \vdash e_2 : \texttt{int}}{\rho \vdash e_1 < e_2 : \texttt{bool}}$$

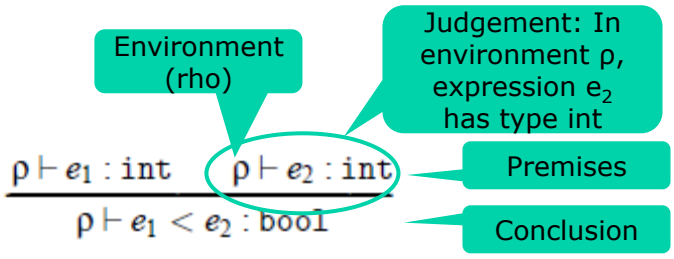
$$\frac{\rho \vdash e_r : t_r \qquad \rho[x \mapsto t_r] \vdash e_b : t}{\rho \vdash \texttt{let } x = e_r \texttt{ in } e_b \texttt{ end } : t}$$

$$\frac{\rho \vdash e_1 : \texttt{bool} \qquad \rho \vdash e_2 : t \qquad \rho \vdash e_3 : t}{\rho \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : t}$$

$$\frac{\rho[x \mapsto t_x, f \mapsto t_x \to t_r] \vdash e_r : t_r \qquad \rho[f \mapsto t_x \to t_r] \vdash e_b : t}{\rho \vdash \texttt{let } f\ (x : t_x) = e_r : t_r \texttt{ in } e_b : t}$$

$$\frac{\rho(f) = t_x \to t_r \qquad \rho \vdash e : t_x}{\rho \vdash f\ e : t_r}$$

How to read a type rule



- IF
  - in environment ρ, expression e_1 has type int, and
  - in environment ρ, expression e_2 has type int
- THEN
  - environment ρ, expression $e_1 < e_2$ has type bool

Joint exercise: How read these?

$\rho \vdash i : \text{int}$

An integer constant has type int

$$\frac{\rho(x) = t}{\rho \vdash x : t}$$

$$\frac{\rho \vdash e_1 : \text{bool} \qquad \rho \vdash e_2 : t \qquad \rho \vdash e_3 : t}{\rho \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : t}$$

$$\frac{\rho \vdash e_r : t_r \qquad \rho[x \mapsto t_r] \vdash e_b : t}{\rho \vdash \text{let } x = e_r \text{ in } e_b \text{ end } : t}$$

Combining type rules to trees

- Stacking type rules on top of each other
- One rule's conclusion is another's premise
- Checking **`let x=1 in x<2 end : bool`** in some environment ρ:

$$\frac{\rho \vdash 1 : \text{int} \qquad \dfrac{\rho[x \mapsto \text{int}] \vdash x : \text{int} \qquad \rho[x \mapsto \text{int}] \vdash 2 : \text{int}}{\rho[x \mapsto \text{int}] \vdash x < 2 : \text{bool}}}{\rho \vdash \text{let } x = 1 \text{ in } x < 2 \text{ end} : \text{bool}}$$

- The **`typ`** function implements the rules, from conclusion to premise!

Joint exercises: Invent type rules

- For e_1 `&&` e_2 (logical and)
- For e_1 `::` e_2 (list cons operator)
- For `match e with [] ->` e_1 `| x::xr ->` e_2

Dynamically or statically typed

- Dynamically typed:
  - Types are checked during evaluation (micro-ML, Postscript, JavaScript, Python, Ruby, Scheme, …)

```
true { 11 } { 22 false add } ifelse =
```

OK, gives 11

- Statically typed:
  - Types are checked before evaluation (our typed fun. language, F#, most of Java and C#)

```
if true then 11 else 22+false
```

Type error

```
true ? 11 : (22 + false)
```

Type error

Dynamic typing in Java/C# arrays

- For a Java/C# array whose element type is a reference type, all assignments are type-checked at runtime

```
void M(Object[] arr, Object x) {
  arr[0] = x;
}
```

Type check needed at run-time

- Why is that necessary?

```
String[] ss = new String[1];
M(ss, new Object());
String s0 = ss[0];
```


Reading and homework

- This week's lecture:
  - PLCSD chapter 4
  - Mogensen 2010 sections 3.12, 3.17
  - Exercises 4.1, 4.2, 4.3, 4.4, 4.5 for Tue 27 Sep
- Next week's lecture:
  - PLCSD chapter 5.1-5.4 and chapter 6