Lexing and Parsing

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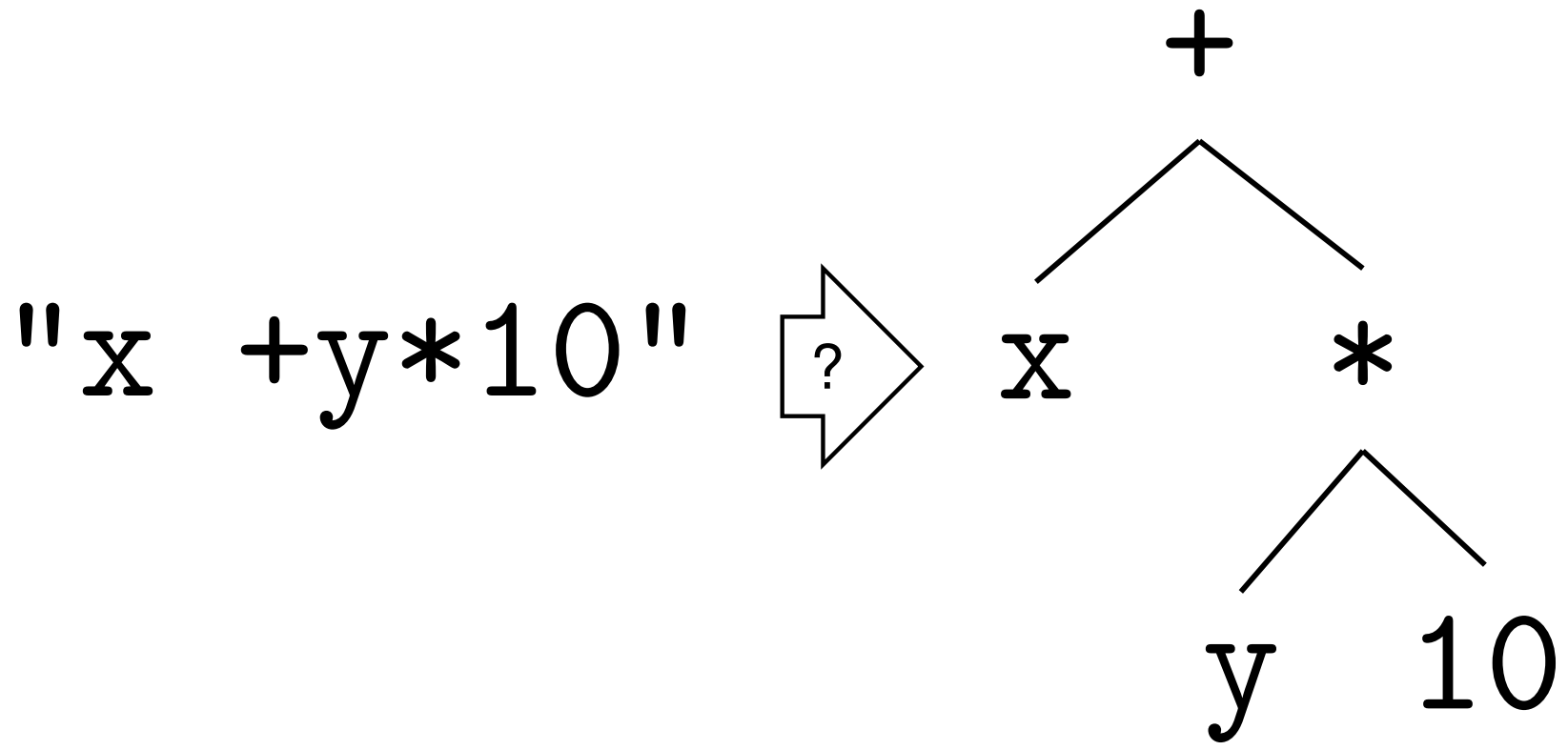
2 September, 2013

Based on slides by Peter Sestoft

IT University
of Copenhagen

From text file to abstract syntax



From text file to abstract syntax

```
"x +y*10"
```

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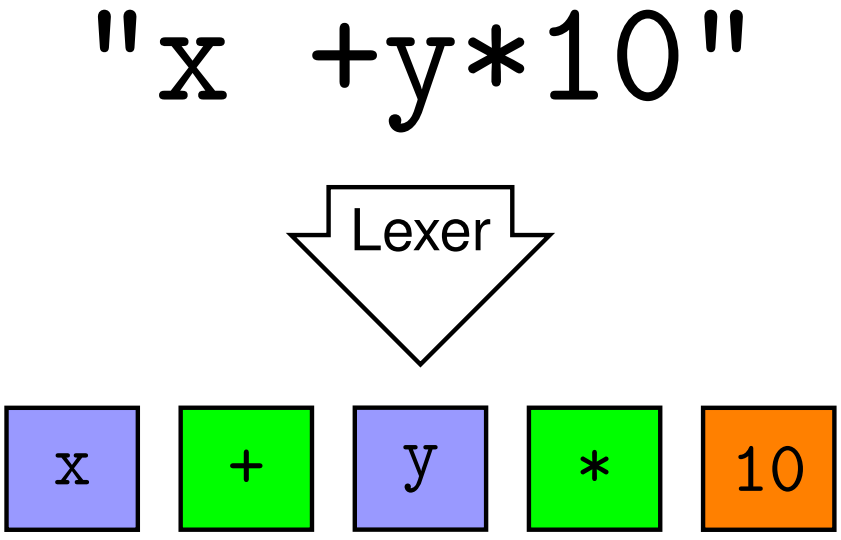

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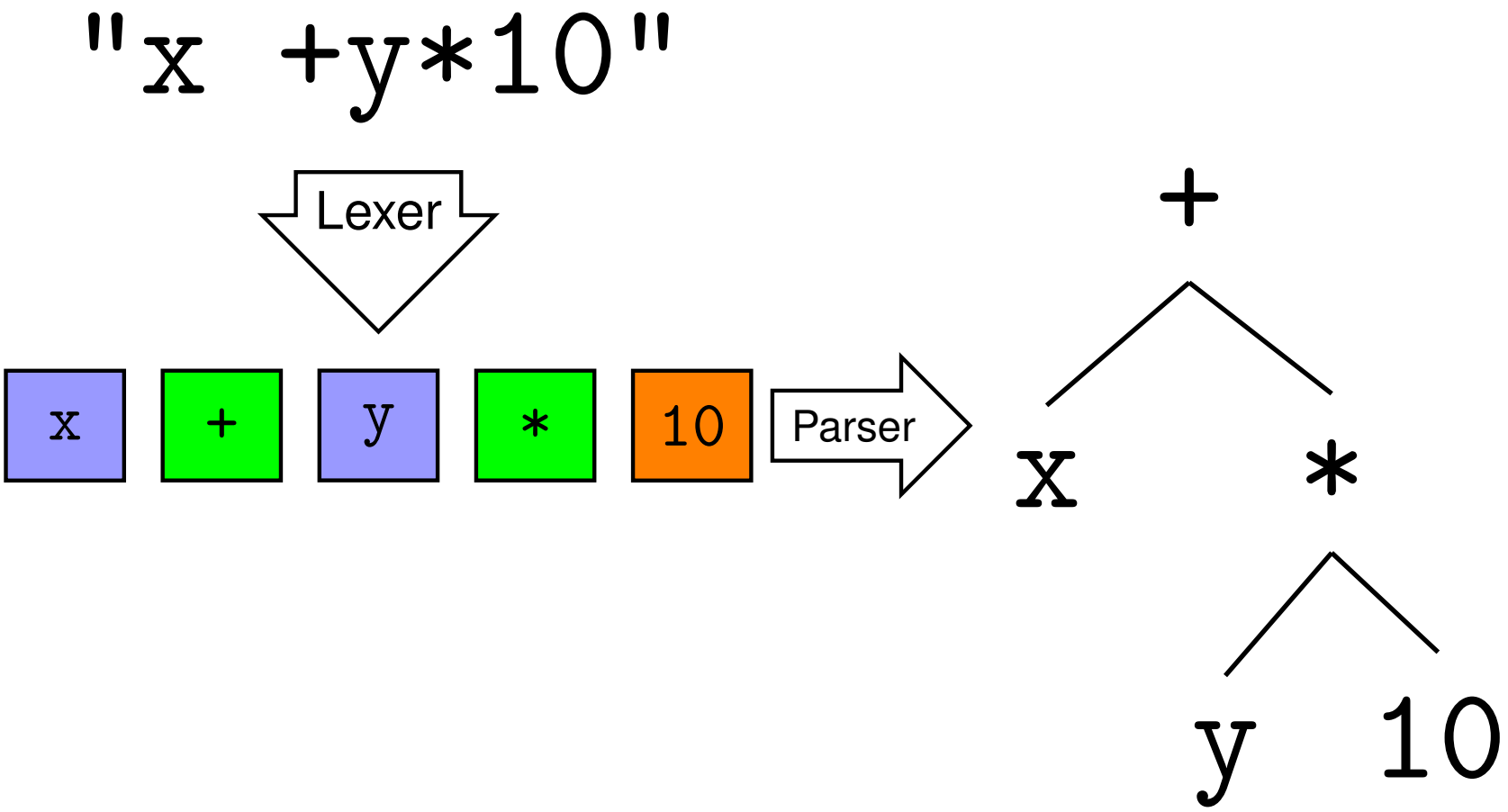

Plan for today

Lexer specifications

- Regular expressions
- The fslex lexer generation tool
- Automata

Parser specifications

- Grammars
- Parsing
- The fsyacc parser generation tool

Parsing Algorithms

- Top-down
- Bottom-up

Languages and automata

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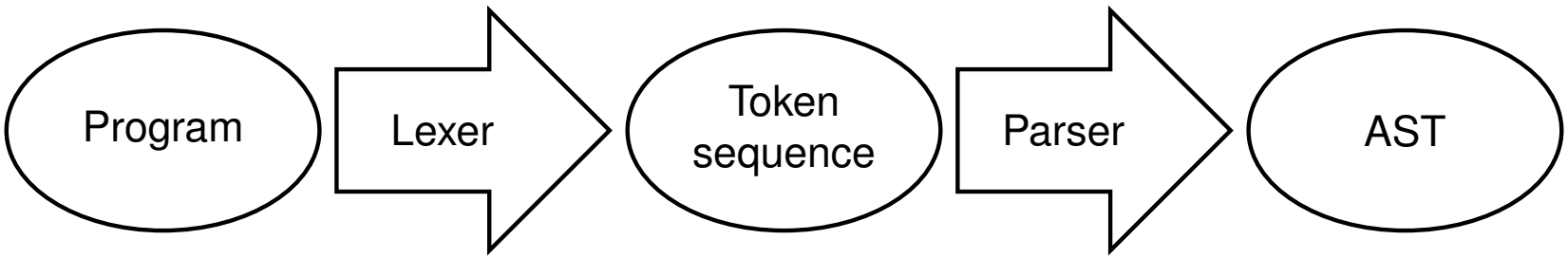
Languages and automata

Lexers and lexer generators



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EXAMPLES

$\texttt{ab}^*$ represents $\{"\texttt{a}", "\texttt{ab}", "\texttt{abb}", \ldots\}$

$(\texttt{ab})^*$ represents $\{"", "\texttt{ab}", "\texttt{abab}", \ldots\}$

$(\texttt{a}|\texttt{b})^*$ represents $\{"", "\texttt{a}", "\texttt{b}", "\texttt{aa}", "\texttt{ab}", "\texttt{ba}", \ldots\}$

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Examples

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Exercise

What does $(\texttt{a}|\texttt{b})\texttt{c}^*$ represent?

Regular expression abbreviations

Abbrev.	Meaning	Expansion
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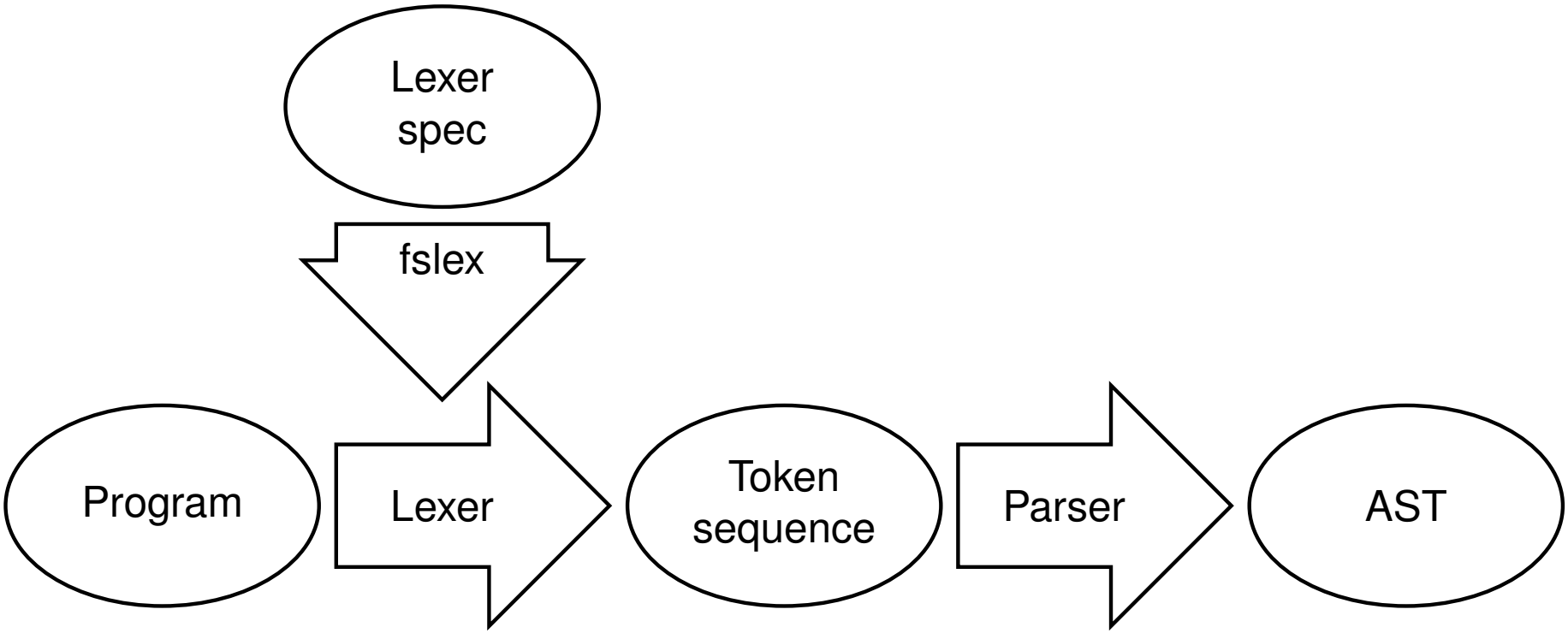
Five-minute exercises

Write regular expressions for:

- Non-negative integer constants
- Integer constants
- Floating-point constants:
  - `3.14`
  - `3E8`
  - `+6.02E23`
- Java variable names:
  - `xy`
  - `x12`
  - `_x`
  - `$x12`

Lexer specification and generator



Lexer specifications: ExprLex.fsl

```
rule Token = parse
  | [' ' '\t' '\n' '\r']  { Token lexbuf }
  | ['0'-'9']+            { CSTINT (...) }
  | ['a'-'z''A'-'Z']['a'-'z''A'-'Z''0'-'9']*
                          { keyword (...) }
  | '+'                   { PLUS  }
  | '-'                   { MINUS }
  | '*'                   { TIMES }
  | '('                   { LPAR  }
  | ')'                   { RPAR  }
  | eof                   { EOF   }
  | _                     { lexerError lexbuf "Bad char" }
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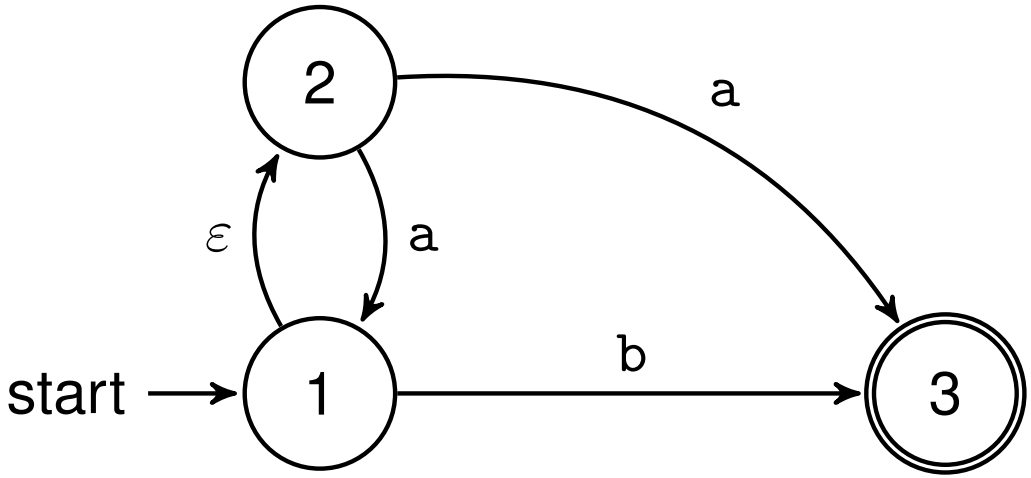
```

F# to construct token

Finite State Automata



Regular expressions and finite automata

For every regular expression *r*, there exists a deterministic finite automaton that recognizes precisely the strings described by *r*.

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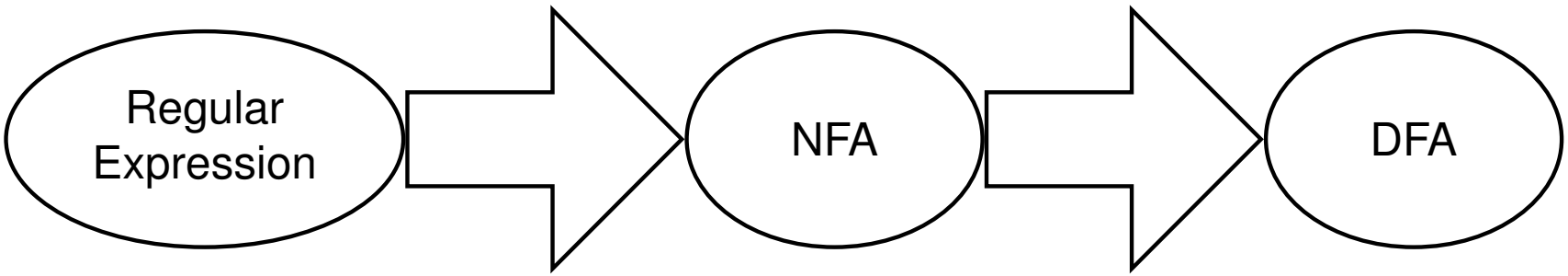
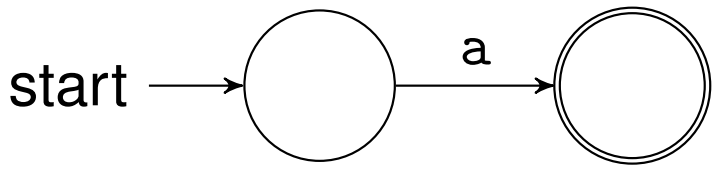
(The converse is also true.)

Constructing automata from regular expressions



Constructing NFA from RE

a



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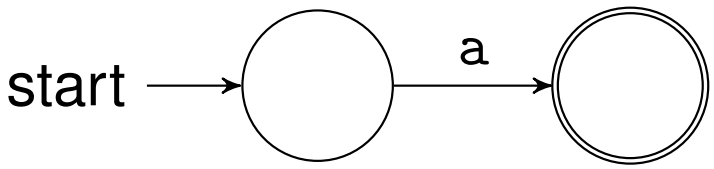
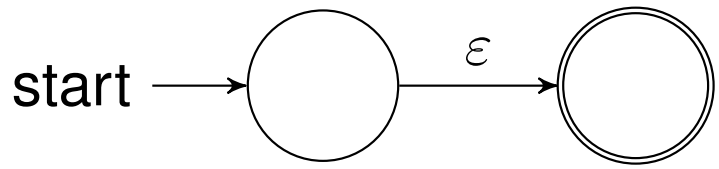
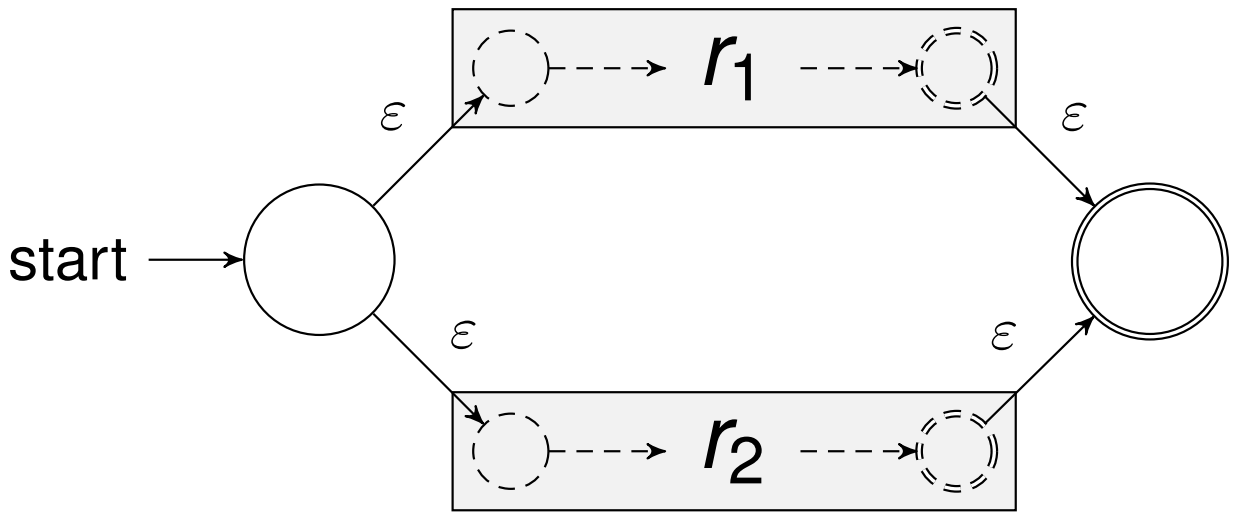


ε



RE to NFA: Alternatives

$$r_1 | r_2$$



RE to NFA: Sequencing

$r_1 r_2$

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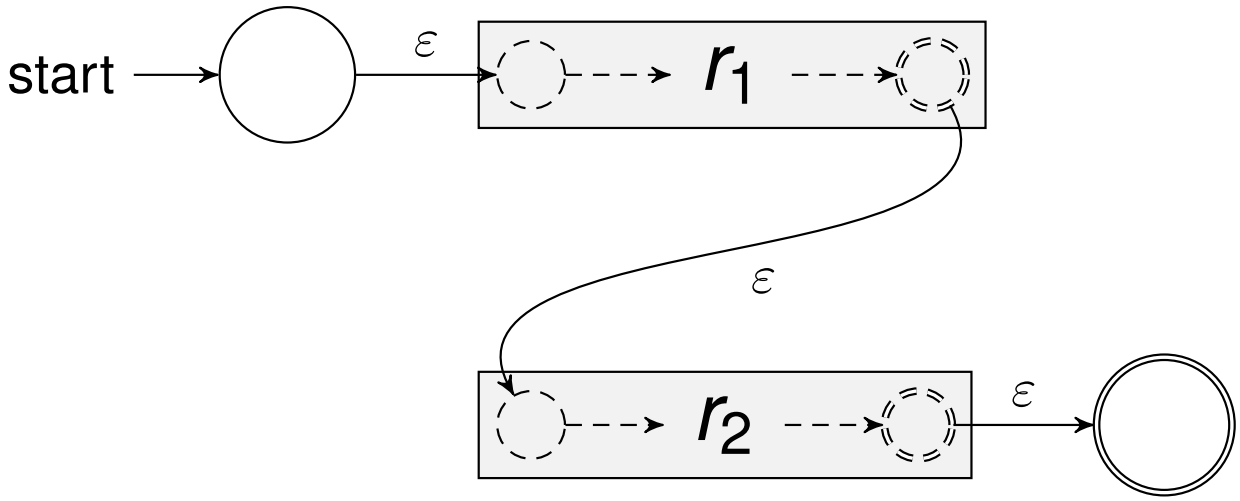
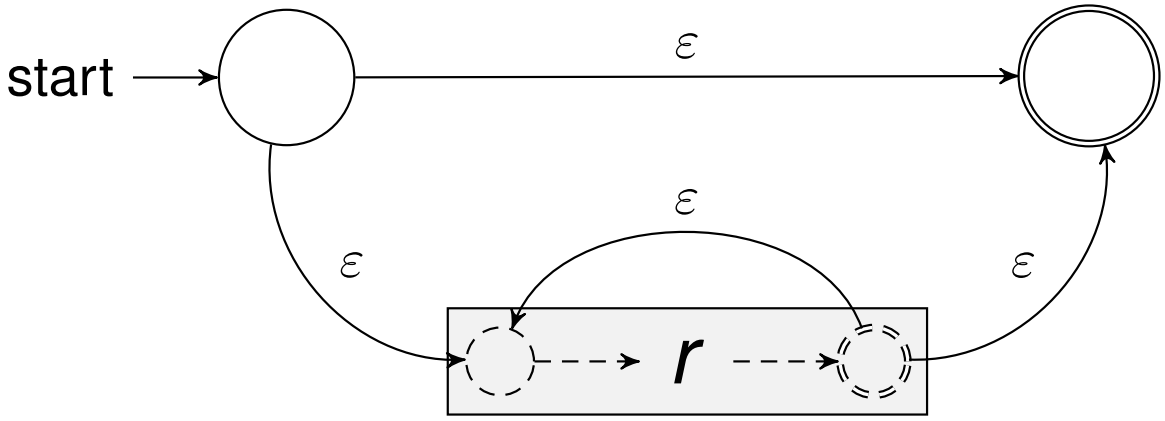


RE to NFA: Repetition

r^*

RE to NFA: Repetition

r^*



Deterministic Finite Automata

- No ε-transitions
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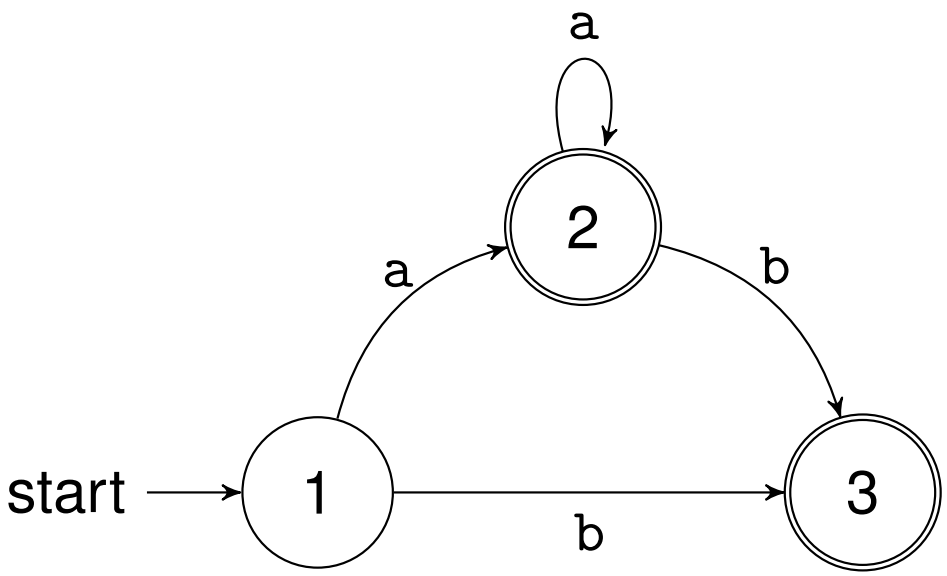

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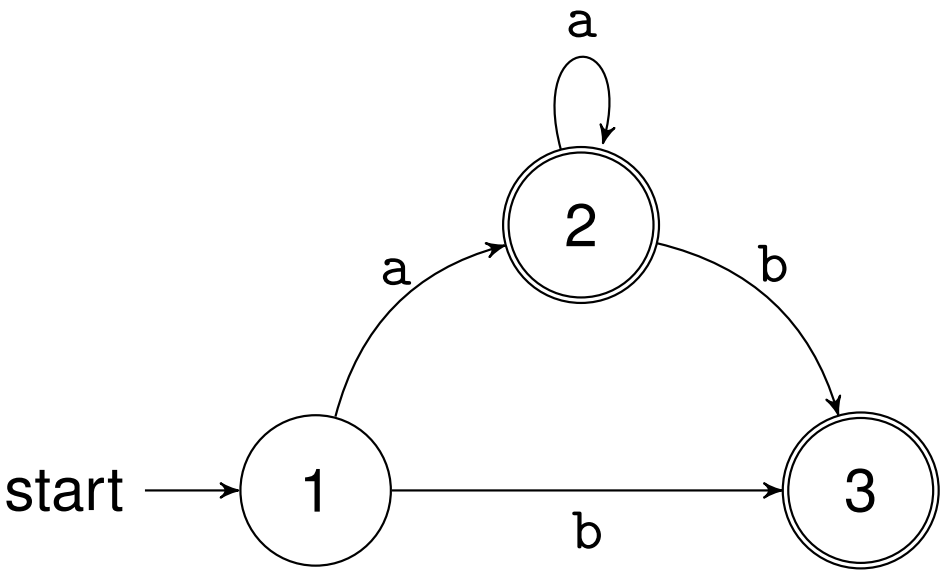
```
state = table[state][input];
```



State	Input	Go to
1	a	2
1	b	3
2	a	2
2	b	3
3	a	fail
3	b	fail

NFA to DFA

For every NFA there is a corresponding DFA that accepts the same set of strings.

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States

The ε-closed sets of NFA states. They are accepting if they contain an accepting NFA state.

Transitions

For every NFA transition from q_0 to q_1 on $\mathtt{a}$, the DFA has a transition from each state containing q_0 to each state containing q_1 on $\mathtt{a}$.

Whiteboard: NFA to DFA demos

- $(ab)^*$

Whiteboard: NFA to DFA demos

- $(\mathtt{ab})^*$
- $(\mathtt{a}|\mathtt{b})^*$

Whiteboard: NFA to DFA demos

- $(ab)^*$
- $(a|b)^*$
- $a^*(a|b)aa$

(Exercise 1.2 from Mogensen's ICD 2011)

Break

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Languages and automata

Context-free grammars

```
Main ::= Expr EOF                                (rule A)
Expr ::= NAME                                    (rule B)
       | CSTINT                                  (rule C)
       | - CSTINT                                (rule D)
       | ( Expr )                                (rule E)
       | let NAME = Expr in Expr end             (rule F)
       | Expr *  Expr                            (rule G)
       | Expr + Expr                             (rule H)
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- Terminals (from lexer)
- Productions (called A–H)
- Start symbol (the nonterminal Main)

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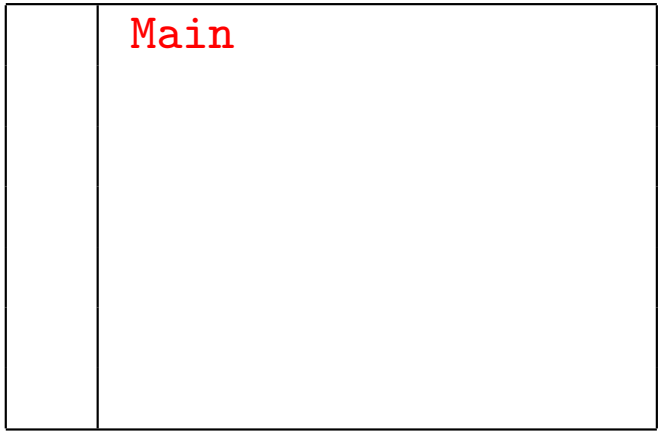
Derivation: grammar as string generator

Main



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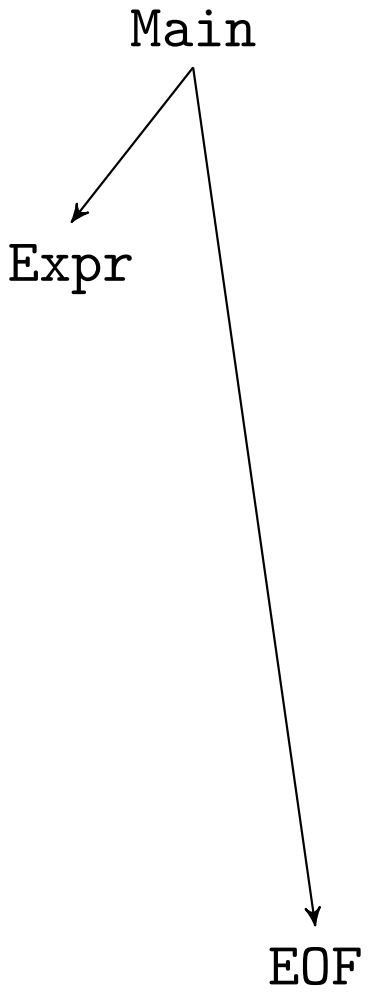

	Main
A	Expr EOF

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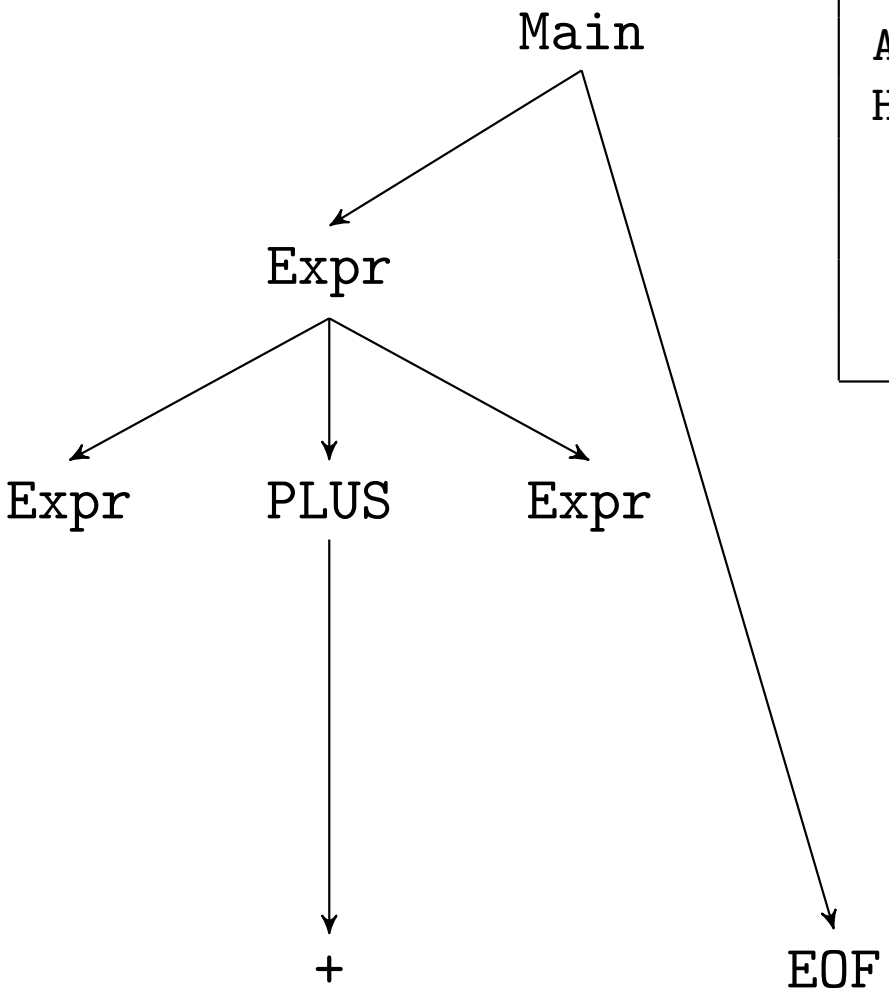
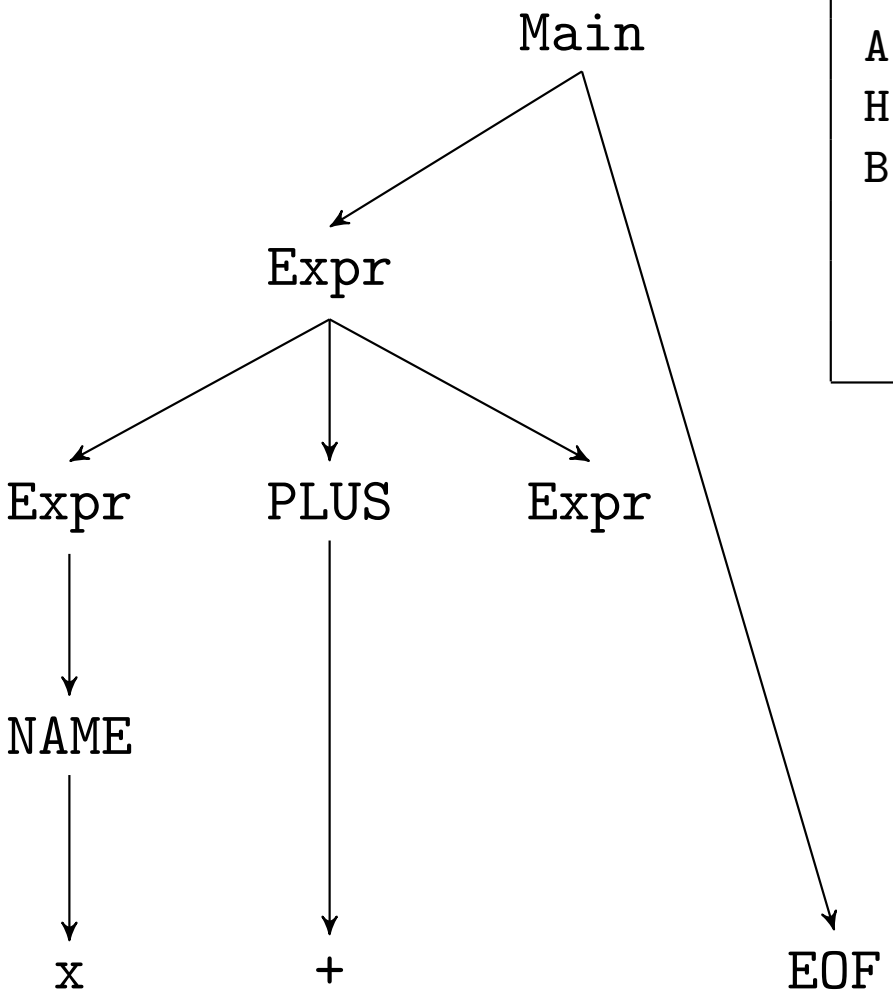


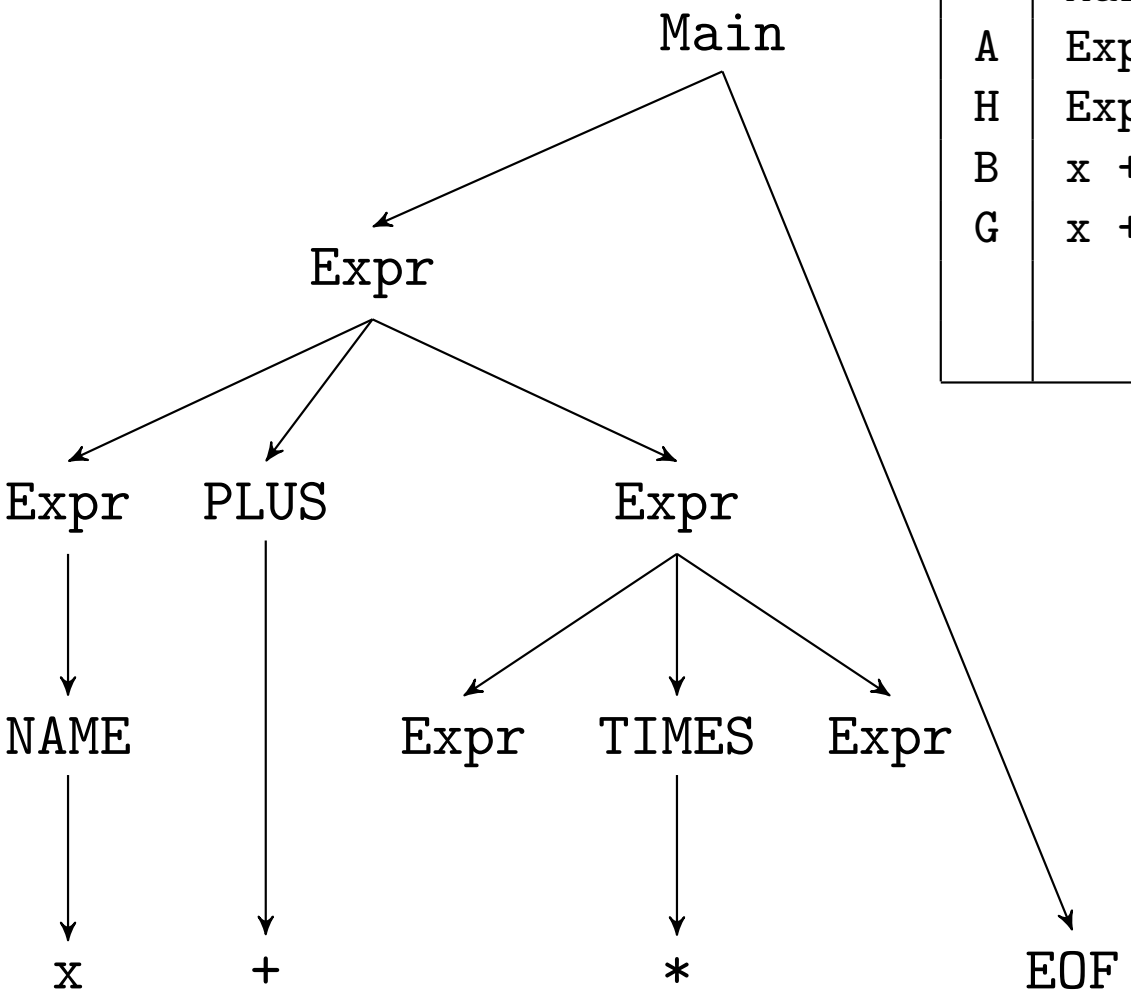
	Main
A	Expr EOF
H	Expr + Expr EOF

Derivation: grammar as string generator



	Main
A	Expr EOF
H	Expr + Expr EOF
B	x + Expr EOF

Derivation: grammar as string generator



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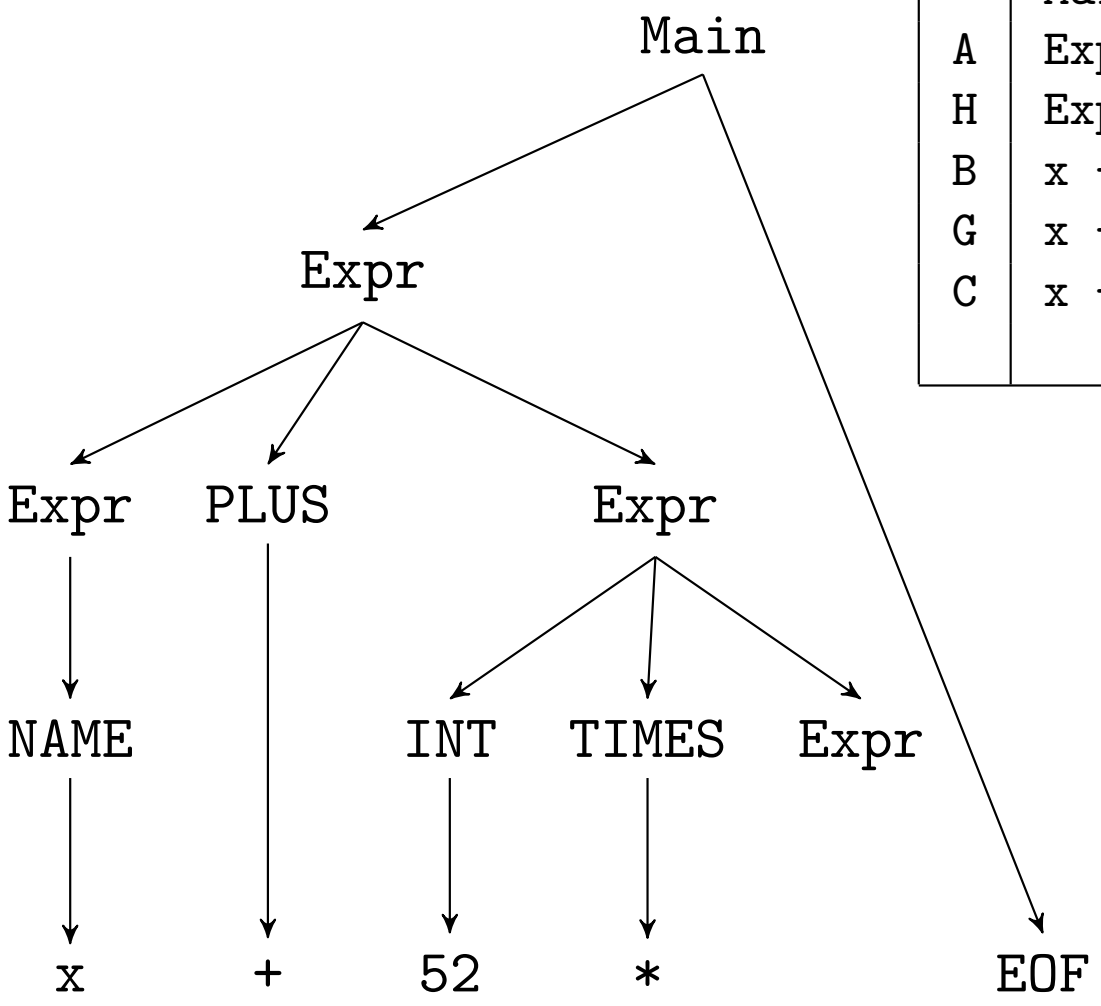
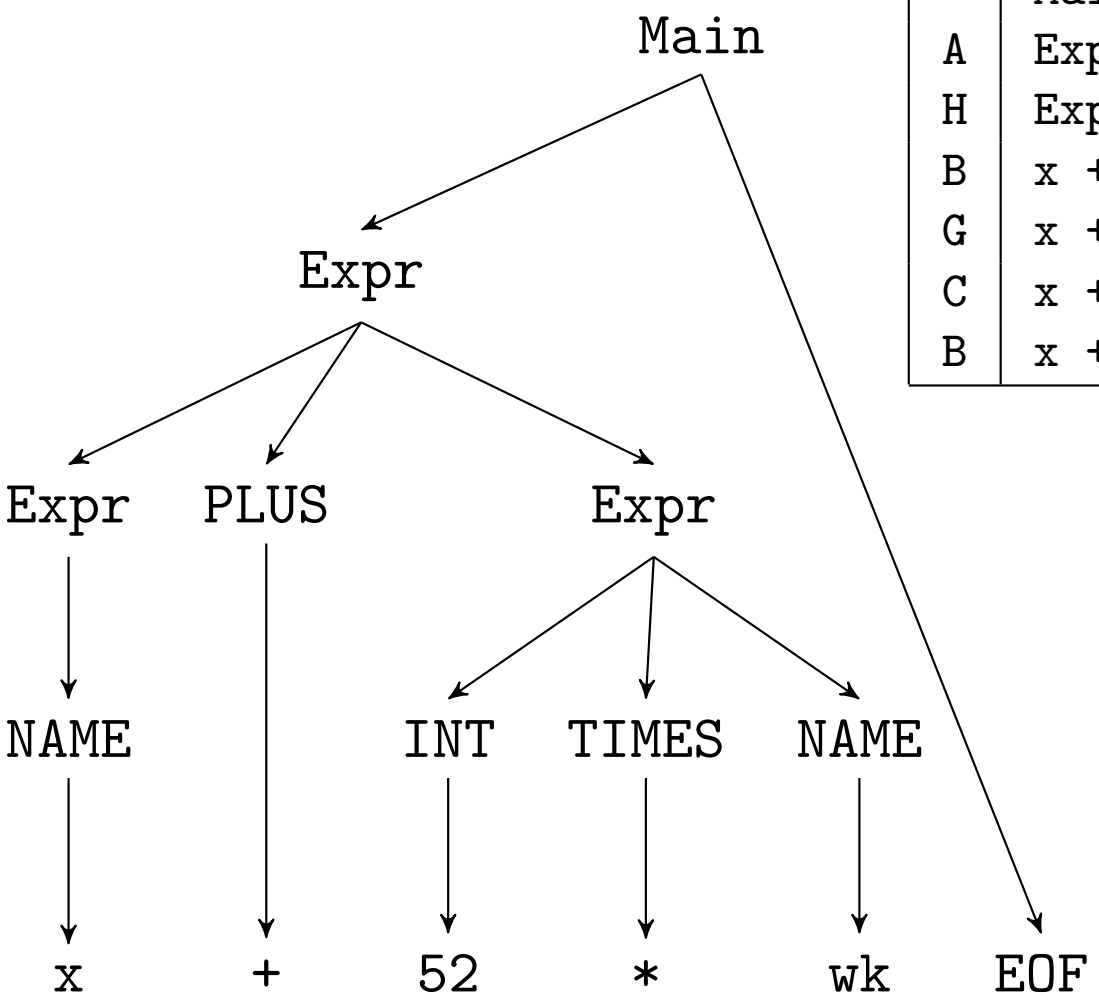
A	Expr EOF
H	Expr + Expr EOF
B	x + Expr EOF
G	x + Expr * Expr EOF

Derivation: grammar as string generator



	Main
A	Expr EOF
H	Expr + Expr EOF
B	x + Expr EOF
G	x + Expr * Expr EOF
C	x + 52 * EXPR EOF

Derivation: grammar as string generator



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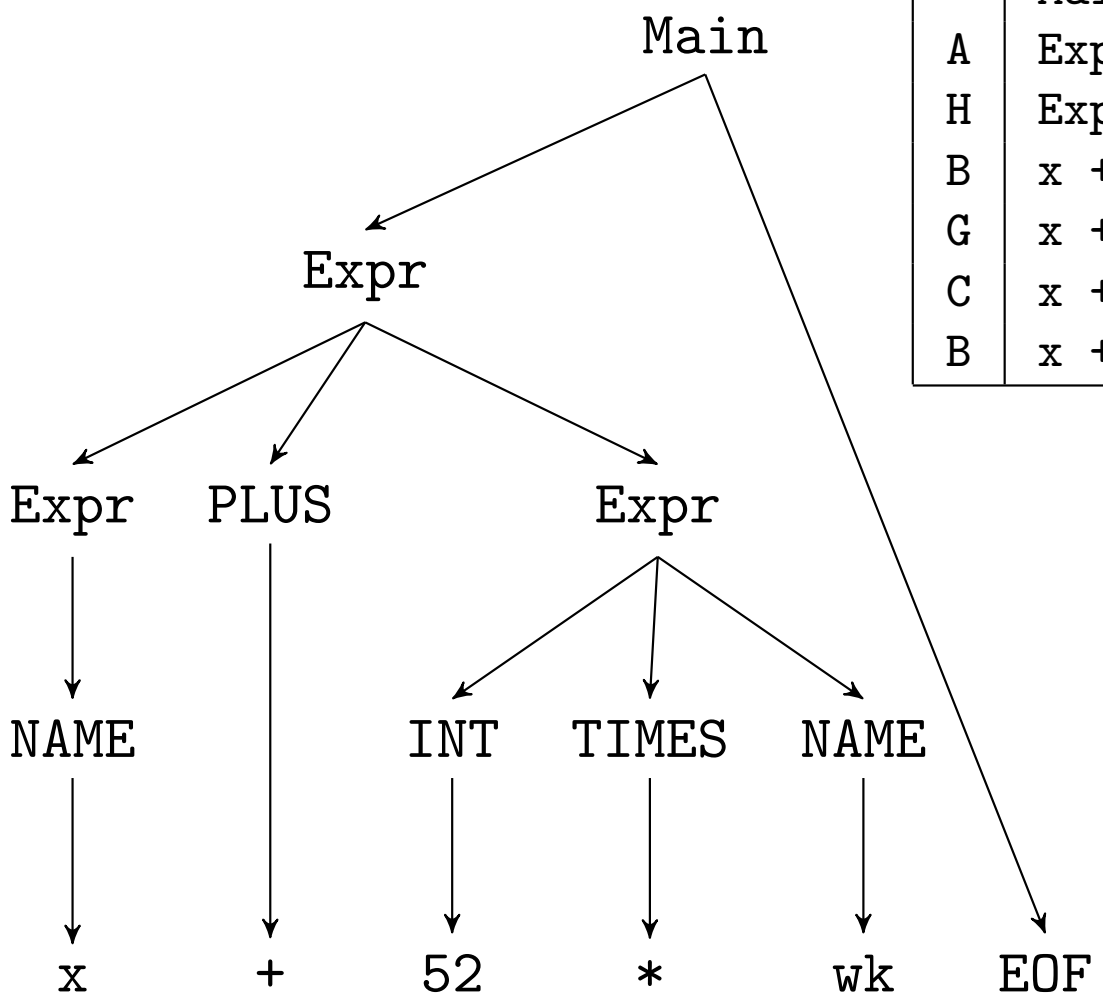

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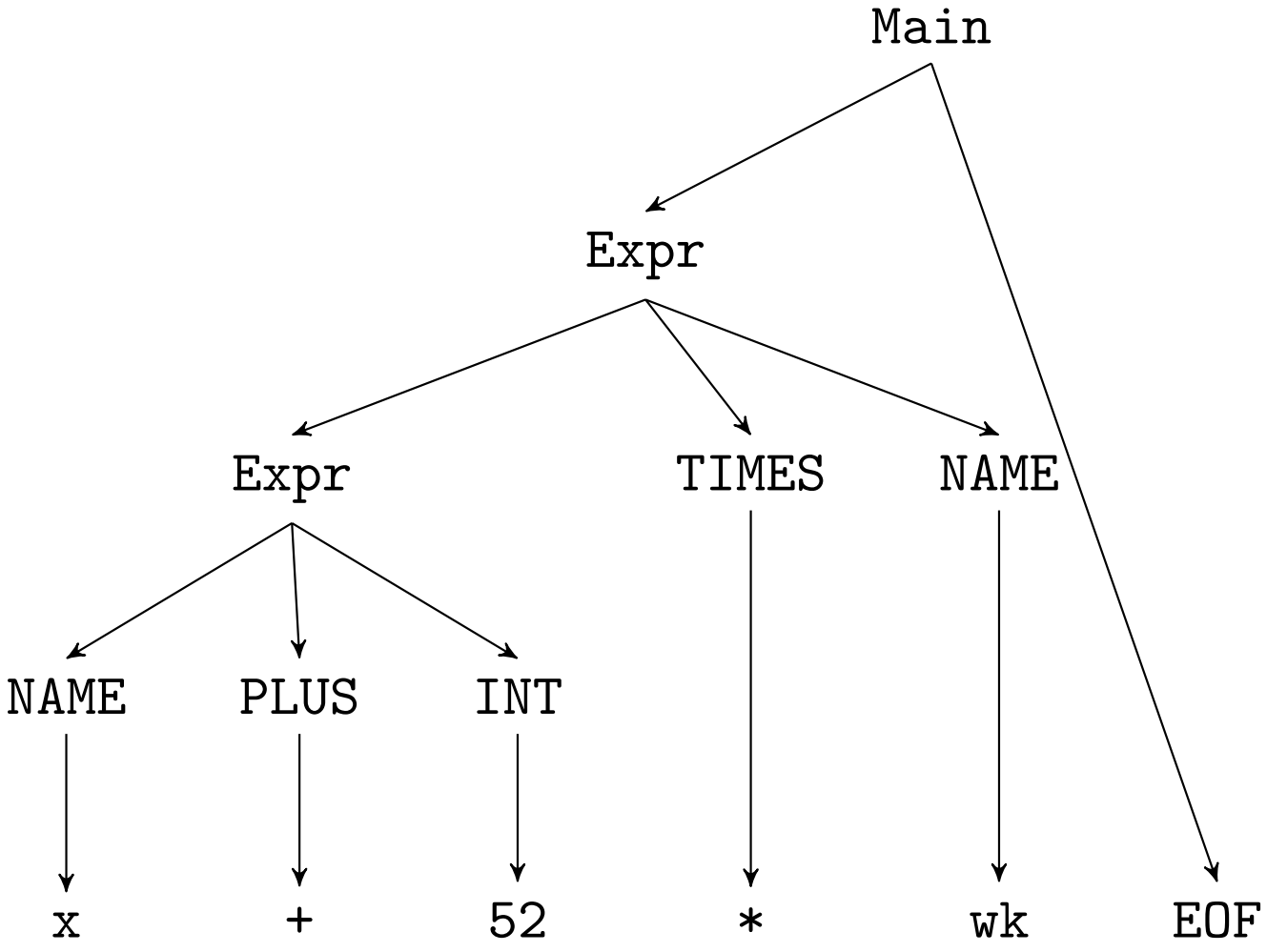


Leftmost and rightmost derivations

Leftmost derivation

Always expand the leftmost nonterminal.
See first example.

Rightmost derivation

Always expand the rightmost nonterminal.
See second example.

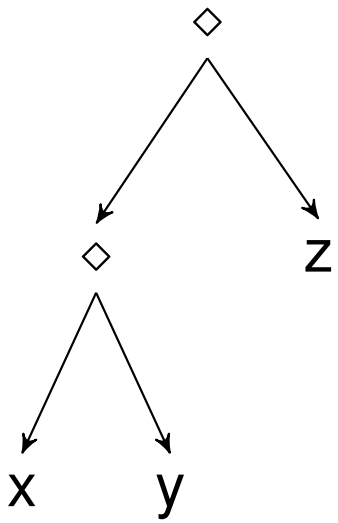
Associativity

How to read $x \diamond y \diamond z$?

Associativity

How to read $x \diamond y \diamond z$?

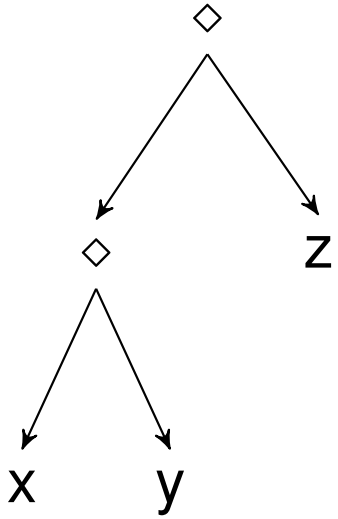
$\diamond$ is left-associative



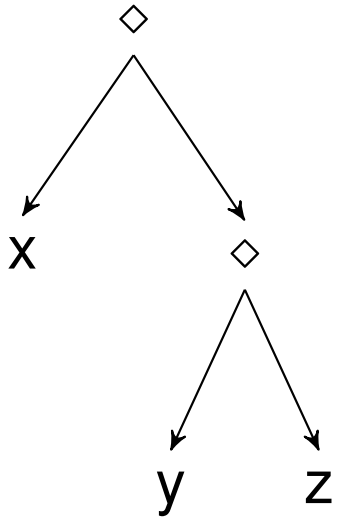
Associativity

How to read $x \diamond y \diamond z$?

$\diamond$ is left-associative



$\diamond$ is right-associative



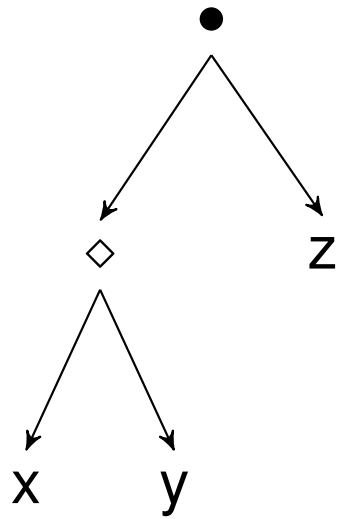
Precedence

How to read $x \diamond y \bullet z$?

Precedence

How to read $x \diamond y \bullet z$?

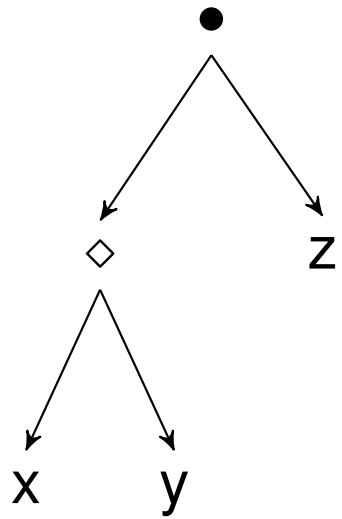
- $\diamond$ has higher precedence



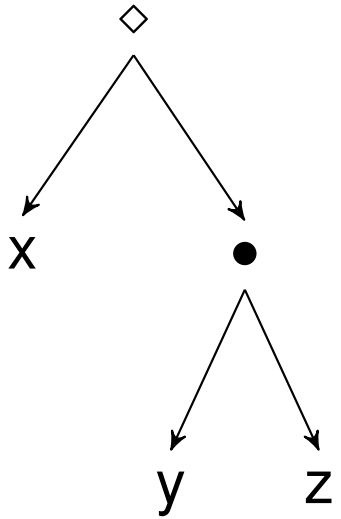
Precedence

How to read $x \diamond y \bullet z$?

$\diamond$ has higher precedence



$\bullet$ has higher precedence



Exercise

What Java or C# operators

- are left-associative?
- are right-associative?
- have higher or lower precedence than others?

Java operator precedence

`() [] .`	Left
`x++ x--`	Right
`++x --x +x -x !x ~x (T)x`	Right
`* / %`	Left
`+ -`	Left
`<< >>`	Left
`< <= > >= instanceof`	Left
`== !=`	Left
`&`	Left
`^`	Left
`|`	Left
`&&`	Left
`||`	Left
`b ? tt : ff`	Right
`= += -= *= /= %= &= ^= |= <<= >>= >>>=`	Right

Parsing is inverse derivation

PARSING

Reconstruct the derivation for a string, if possible

Parsing is inverse derivation

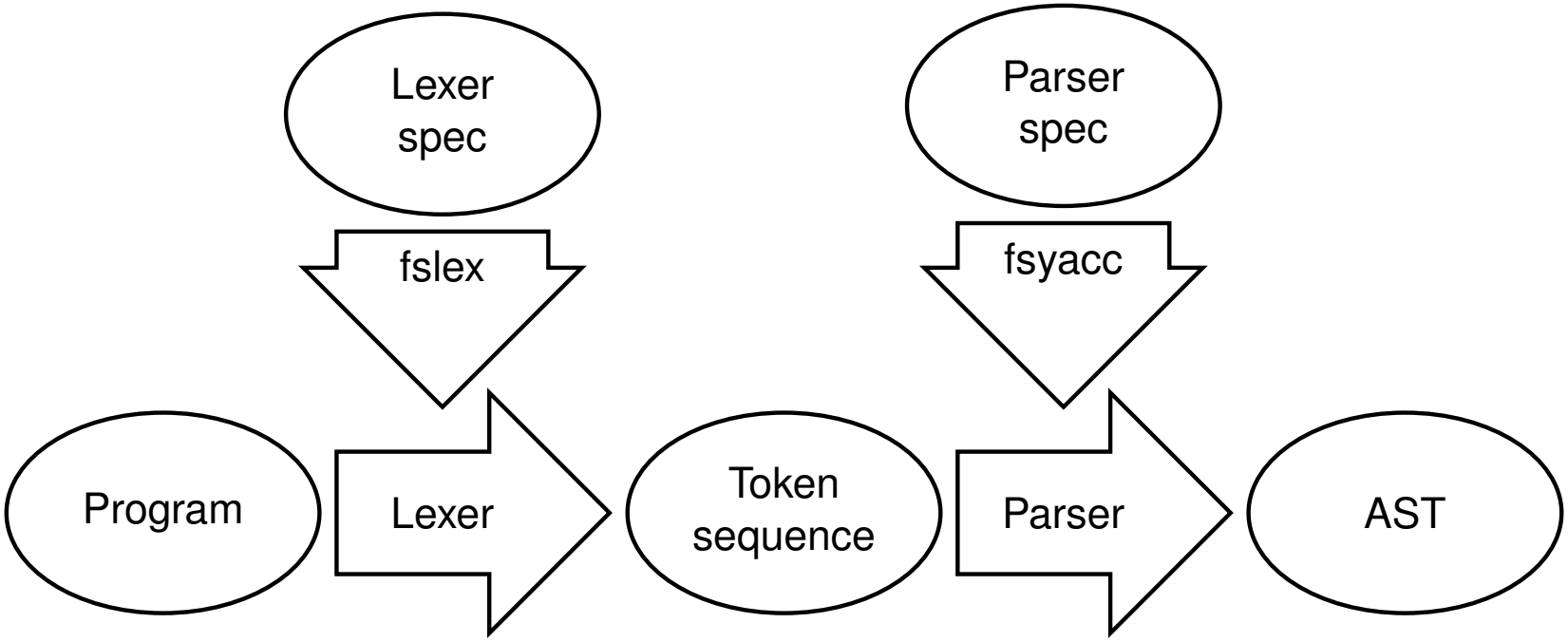
Parsing

Reconstruct the derivation for a string, if possible

Methods

- Top-down: parser structured like grammar. Example next week.
- Generated bottom-up: parser generated using tool.

Parser specification and generator



Tokens, associativity and precedence in fsyacc

```
%token <int> CSTINT
%token <string> NAME
%token PLUS MINUS TIMES EQ
%token END IN LET
%token LPAR RPAR
%token EOF

%left MINUS PLUS  /* lowest precedence  */
%left TIMES       /* highest precedence */
```

Tokens, associativity and precedence in fsyacc

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```

Token specifications - expanded to a datatype

Tokens, associativity and precedence in fsyacc

```
%token <int> CSTINT
%token <string> NAME
%token PLUS MINUS TIMES EQ
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%token LPAR RPAR
%token EOF

%left MINUS PLUS  /* lowest precedence  */
%left TIMES       /* highest precedence */
```

Tokens carrying data

Tokens, associativity and precedence in fsyacc

```
%token <int> CSTINT
%token <string> NAME
%token PLUS MINUS TIMES EQ
%token END IN LET
%token LPAR RPAR
%token EOF

%left MINUS PLUS  /* lowest precedence  */
%left TIMES       /* highest precedence */
```

Precedence: `%left`, `%right`, and `%nonassoc` allowed

Tokens, associativity and precedence in fsyacc

```
%token <int> CSTINT
%token <string> NAME
%token PLUS MINUS TIMES EQ
%token END IN LET
%token LPAR RPAR
%token EOF

%left MINUS PLUS  /* lowest precedence  */
%left TIMES       /* highest precedence */
```

Ordering of groups defines precedence levels

Parser specification

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                          { $1                 } A
Expr:
    NAME                              { Var $1             } B
  | CSTINT                            { CstI $1            } C
  | MINUS CSTINT                      { CstI (- $2)        } D
  | LPAR Expr RPAR                    { $2                 } E
  | LET NAME EQ Expr IN Expr END      { Let($2, $4, $6)    } F
  | Expr TIMES Expr                   { Prim("*", $1, $3) } G
  | Expr PLUS  Expr                   { Prim("+", $1, $3) } H
  | Expr MINUS Expr                   { Prim("-", $1, $3) } I
```

Parser specification

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                              { $1                  } A
Expr:
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  | Expr PLUS  Expr                       { Prim("+", $1, $3)   } H
  | Expr MINUS Expr                       { Prim("-", $1, $3)   } I
```

Non-terminals

Parser specification

```
%start Main
%type <Absyn.expr> Main
%%
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    Expr EOF                              { $1                  } A
Expr:
    NAME                                  { Var $1              } B
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```

Start symbol

Parser specification

```
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Expr:
    NAME                                  { Var $1              } B
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```

Semantic actions

Parser specification

```
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Main:
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```

Arguments count from left

Parser specification

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%start Main
%type <Absyn.expr> Main
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Main:
    Expr EOF                              { $1                  } A
Expr:
    NAME                                  { Var $1              } B
  | CSTINT                                { CstI $1             } C
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Arguments count from left

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  | Expr TIMES Expr                   { Prim("*", $1, $3)   } G
  | Expr PLUS  Expr                   { Prim("+", $1, $3)   } H
  | Expr MINUS Expr                   { Prim("-", $1, $3)   } I
```

Type annotation

Putting together lexer and parser

From file `Expr/Parse.fs`:

```
let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    in try
         ExprPar.Main ExprLex.Token lexbuf
       with
         | exn -> failwith "Lexing or parsing error ... "
```

Putting together lexer and parser

From file `Expr/Parse.fs`:

```
let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    in try
         ExprPar.Main ExprLex.Token lexbuf
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Entry point in parser

Putting together lexer and parser

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let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    in try
         ExprPar.Main ExprLex.Token lexbuf
       with
         | exn -> failwith "Lexing or parsing error ... "
```

Entry point in lexer

Invoking fslex and fsyacc

- Build the lexer and parser vs files ExprLex.fs and ExprPar.fs
- Compile as modules together with Absyn.fs and Parse.fs:

```
$ fsyacc --module ExprPar ExprPar.fsy
$ fslex --unicode ExprLex.fsl
$ fsi -r FSharp.PowerPack Absyn.fs ExprPar.fs
                          ExprLex.fs Parse.fs
```

- Open the Parse module and experiment:

```
open Parse;;
fromString "x + 52 * wk";;
```

Whiteboard

How do we change the lexer and/or parser to

- accept brackets `[ ]` in addition to parens `( )`?

Whiteboard

How do we change the lexer and/or parser to

- accept brackets `[ ]` in addition to parens `( )`?
- accept the division operator (`/`) also?

Whiteboard

How do we change the lexer and/or parser to

- accept brackets `[ ]` in addition to parens `( )`?
- accept the division operator (`/`) also?
- accept the syntax
  `{ x <- 2 in x * 3 }`
  instead of
  `let x = 2 in x * 3 end`
  ?

Whiteboard

How do we change the lexer and/or parser to

- accept brackets `[ ]` in addition to parens `( )`?
- accept the division operator (`/`) also?
- accept the syntax
  `{ x <- 2 in x * 3 }`
  instead of
  `let x = 2 in x * 3 end`
  ?
- accept function calls such as `max(x, y)`?

Break

Plan for today

Lexer specifications
- Regular expressions
- The fslex lexer generation tool
- Automata

Parser specifications
- Grammars
- Parsing
- The fsyacc parser generation tool

Parsing Algorithms
- Top-down
- Bottom-up

Languages and automata

Parsing Algorithms

LR

Read *Left* to right, make derivations from *Rightmost* nonterminal.

- Build parse tree from bottom to top
- Very difficult to get right, but excellent tools (like `fsyacc`)
- Easy encoding of precedence and associativity

Parsing Algorithms

LR

Read *Left* to right, make derivations from *Rightmost* nonterminal.

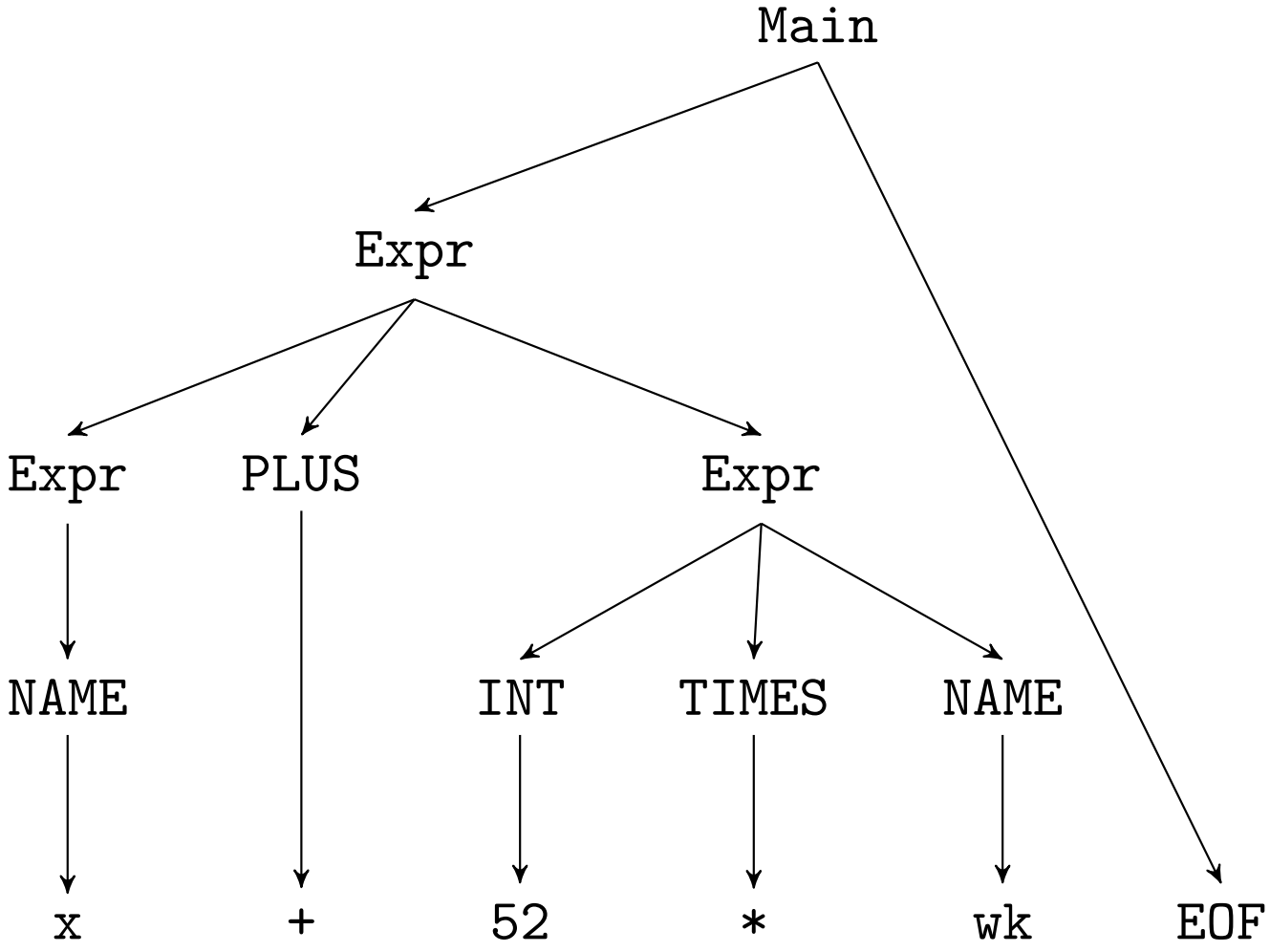
- Build parse tree from bottom to top
- Very difficult to get right, but excellent tools (like `fsyacc`)
- Easy encoding of precedence and associativity

LL

Read *Left* to right, make derivations from *Leftmost* nonterminal.

- Build tree from top to bottom
- Parsing follows structure of grammar — easy to read and write
- Precedence and associativity require modifying grammar

Our derivation — let's parse!



The (fsyacc, LR) parser automaton

(File `ExprPar.fsyacc.output` from `fsyacc -v ExprPar.fsy`)

```
state 19:
     items:
       Expr -> Expr . 'TIMES' Expr
       Expr -> Expr . 'PLUS' Expr
       Expr -> Expr 'PLUS' Expr .
       Expr -> Expr . 'MINUS' Expr
     actions:
       action 'EOF':   reduce Expr --> Expr 'PLUS' Expr
       action 'LPAR':   reduce Expr --> Expr 'PLUS' Expr
       action 'RPAR':   reduce Expr --> Expr 'PLUS' Expr
       action 'END':   reduce Expr --> Expr 'PLUS' Expr
       action 'IN':   reduce Expr --> Expr 'PLUS' Expr
       action 'LET':   reduce Expr --> Expr 'PLUS' Expr
       action 'PLUS':   reduce Expr --> Expr 'PLUS' Expr
       action 'MINUS':   reduce Expr --> Expr 'PLUS' Expr
       action 'TIMES':   shift 21
       action 'EQ':   reduce Expr --> Expr 'PLUS' Expr
       action 'NAME':   reduce Expr --> Expr 'PLUS' Expr
       action 'CSTINT':   reduce Expr --> Expr 'PLUS' Expr
       action 'error':   reduce Expr --> Expr 'PLUS' Expr
       action '#':   reduce Expr --> Expr 'PLUS' Expr
       action '$$':   reduce Expr --> Expr 'PLUS' Expr
     immediate action: <none>
    gotos:
```

- Parser consists of states, transitions, and stack
- Dots in items show current positions
- Actions determine what happens based on input token

Parser stack example

```
Input        Parse stack (top on right)                    Action
-------------------------------------------------------------------
x+52*wk EOF  #0                                            shift #4
```

Parser stack example

```
Input         Parse stack (top on right)                        Action
-------------------------------------------------------------------------
x+52*wk EOF   #0                                                shift #4
+52*wk EOF    #0 x #4                                           reduce by B
```

Parser stack example

```
Input        Parse stack (top on right)                    Action
-------------------------------------------------------------------
x+52*wk EOF  #0                                            shift #4
+52*wk EOF   #0 x #4                                       reduce by B
+52*wk EOF   #0 Expr                                       goto #2
```

Parser stack example

```
Input        Parse stack (top on right)                    Action
-------------------------------------------------------------------
x+52*wk EOF  #0                                            shift #4
+52*wk EOF   #0 x #4                                       reduce by B
+52*wk EOF   #0 Expr                                       goto #2
+52*wk EOF   #0 Expr #2                                    shift #22
```

Parser stack example

```
Input        Parse stack (top on right)                    Action
-------------------------------------------------------------------
x+52*wk EOF  #0                                            shift #4
+52*wk EOF   #0 x #4                                       reduce by B
+52*wk EOF   #0 Expr                                       goto #2
+52*wk EOF   #0 Expr #2                                    shift #22
52*wk EOF    #0 Expr #2 + #22                              shift #5
```

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C

Parser stack example

```
Input        Parse stack (top on right)                    Action
-------------------------------------------------------------------
x+52*wk EOF  #0                                            shift #4
+52*wk EOF   #0 x #4                                       reduce by B
+52*wk EOF   #0 Expr                                       goto #2
+52*wk EOF   #0 Expr #2                                    shift #22
52*wk EOF    #0 Expr #2 + #22                              shift #5
*wk EOF      #0 Expr #2 + #22 52 #5                        reduce by C
*wk EOF      #0 Expr #2 + #22 Expr                         goto #19
```

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
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+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19

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*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
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EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H

Parser stack example

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x+52*wk EOF	#0	shift #4
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*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
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EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2
EOF	#0 Expr #2	shift 3

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2
EOF	#0 Expr #2	shift 3
	#0 Expr #2 EOF #3	reduce by A

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2
EOF	#0 Expr #2	shift 3
	#0 Expr #2 EOF #3	reduce by A
	#0 Main	goto #1

Parser stack example

Input	Parse stack (top on right)	Action
x+52*wk EOF	#0	shift #4
+52*wk EOF	#0 x #4	reduce by B
+52*wk EOF	#0 Expr	goto #2
+52*wk EOF	#0 Expr #2	shift #22
52*wk EOF	#0 Expr #2 + #22	shift #5
*wk EOF	#0 Expr #2 + #22 52 #5	reduce by C
*wk EOF	#0 Expr #2 + #22 Expr	goto #19
*wk EOF	#0 Expr #2 + #22 Expr #19	shift #21
wk EOF	#0 Expr #2 + #22 Expr #19 * #21	shift #4
EOF	#0 Expr #2 + #22 Expr #19 * #21 wk #4	reduce by B
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr	goto #18
EOF	#0 Expr #2 + #22 Expr #19 * #21 Expr #18	reduce by G
EOF	#0 Expr #2 + #22 Expr	goto #19
EOF	#0 Expr #2 + #22 Expr #19	reduce by H
EOF	#0 Expr	goto #2
EOF	#0 Expr #2	shift 3
	#0 Expr #2 EOF #3	reduce by A
	#0 Main	goto #1
	#0 Main #1	accept

Parser state and actions

Parser state

Stack of symbols and states

Parser actions

- **Shift** read a symbol from input onto the stack, and go to new state
- **Reduce** take grammar rule RHS symbols off the stack and replace them by its LHS nonterminal, and evaluate a semantic action
- **Goto** go to a new parser state (after reduce)

Shift/reduce conflicts

- Sometimes the parser generator does not know whether to shift or to reduce
  (especially if the grammar is ambiguous)
- Warnings are issued by `fsyacc`

Shift/reduce conflicts

- Sometimes the parser generator does not know whether to shift or to reduce
  (especially if the grammar is ambiguous)
- Warnings are issued by `fsyacc`

- To resolve shift/reduce conflicts, change the parser specification
- To understand how, study the parser automaton in `ExprPar.fsyacc.output`

LL parsing: recursive descent

Scheme terms (S-expressions)

Symbols like `foo`, `bar`, `b52`, `+`, `*`

Numbers like `117`, `-4`

Lists in parens: `(foo (+ n 1))`

LL parsing: recursive descent

Scheme terms (S-expressions)

Symbols like `foo`, `bar`, `b52`, `+`, `*`

Numbers like `117`, `-4`

Lists in parens: `(foo (+ n 1))`

Grammar

```
Sexp ::= SYMBOL
       | NUMBER
       | ( Sexp* )
```

Handwritten lexer and parser in C#

```
interface IToken { }
class Lpar : IToken { ... }
class Rpar : IToken { ... }
class Symbol : IToken {
  public readonly String name;
  ...
}
class NumberCst : IToken {
  public readonly int val;
  ...
}
```

Handwritten lexer and parser in C#

```
public static IEnumerator<IToken> Tokenize(TextReader rd) {
  for (;;) {
    int raw = rd.Read();
    char ch = (char)raw;
    if (raw == -1)
      yield break;
    else if (Char.IsWhiteSpace(ch)) { }
    else if (Char.IsDigit(ch))
      yield return new NumberCst(ScanNumber(ch, rd));
    else switch (ch) {
      case '(':
        yield return Lpar.LPAR; break;
      case ')':
        yield return Rpar.RPAR; break;
      case '-': // negative number, or symbol
        ...
      default:
        yield return ScanSymbol(ch, rd);
        break;
} } }
```

Parsing S-expressions top-down

```
Sexp ::= SYMBOL
       | NUMBER
       | ( Sexp* )
```

- If next token is symbol, success
- If next token is number, success
- If next token is left paren, parse repeated S-expressions while next token not right paren

Handwritten recursive descent parser

```
public static void ParseSexp(IEnumerator<IToken> ts) {
  if (ts.Current is Symbol) {
    Console.WriteLine("Parsed symbol " + ts.Current);
  } else if (ts.Current is NumberCst) {
    Console.WriteLine("Parsed number " + ts.Current);
  }
```

Handwritten recursive descent parser

```
public static void ParseSexp(IEnumerator<IToken> ts) {
  if (ts.Current is Symbol) {
    Console.WriteLine("Parsed symbol " + ts.Current);
  } else if (ts.Current is NumberCst) {
    Console.WriteLine("Parsed number " + ts.Current);
  } else if (ts.Current is Lpar) {
    Console.WriteLine("Started parsing list");
    Advance(ts);
    while (!(ts.Current is Rpar)) {
      ParseSexp(ts);
      Advance(ts);
    }
    Console.WriteLine("Ended parsing list");
  }
```

Handwritten recursive descent parser

```
public static void ParseSexp(IEnumerator<IToken> ts) {
  if (ts.Current is Symbol) {
    Console.WriteLine("Parsed symbol " + ts.Current);
  } else if (ts.Current is NumberCst) {
    Console.WriteLine("Parsed number " + ts.Current);
  } else if (ts.Current is Lpar) {
    Console.WriteLine("Started parsing list");
    Advance(ts);
    while (!(ts.Current is Rpar)) {
      ParseSexp(ts);
      Advance(ts);
    }
    Console.WriteLine("Ended parsing list");
  } else
    throw new ArgumentException("Parse error);
}
```

Handwritten recursive descent parser

```
public static void ParseSexp(IEnumerator<IToken> ts) {
  if (ts.Current is Symbol) {
    Console.WriteLine("Parsed symbol " + ts.Current);
  } else if (ts.Current is NumberCst) {
    Console.WriteLine("Parsed number " + ts.Current);
  } else if (ts.Current is Lpar) {
    Console.WriteLine("Started parsing list");
    Advance(ts);
    while (!(ts.Current is Rpar)) {
      ParseSexp(ts);
      Advance(ts);
    }
    Console.WriteLine("Ended parsing list");
  } else
    throw new ArgumentException("Parse error);
}
...
private static void Advance(IEnumerator<IToken> ts) {
  if (!ts.MoveNext())
    throw new ArgumentException("Unxpected eof");
}
```

Plan for today

Lexer specifications
- Regular expressions
- The fslex lexer generation tool
- Automata

Parser specifications
- Grammars
- Parsing
- The fsyacc parser generation tool

Parsing Algorithms
- Top-down
- Bottom-up

Languages and automata

The Chomsky Hierarchy (1958)

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Type 3: regular grammars

Same expressiveness as regular expressions.

$$A \to cB \qquad A \to B \qquad A \to c \qquad A \to \varepsilon$$

The Chomsky Hierarchy (1958)

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Type 2: Context-free grammars

$$A \to cBd$$

The Chomsky Hierarchy (1958)

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$$A \to cBd$$

Type 1: Context-sensitive grammars

Non-abbreviating rules.

$$aAb \to acAdb$$

The Chomsky Hierarchy (1958)

Type 3: regular grammars

Same expressiveness as regular expressions.

$$A \to cB \qquad A \to B \qquad A \to c \qquad A \to \varepsilon$$

Type 2: Context-free grammars

$$A \to cBd$$

Type 1: Context-sensitive grammars

Non-abbreviating rules.

$$aAb \to acAdb$$

Type 0: unrestricted grammars

Same as term-rewrite systems.

$$0Ay \to 0$$

Chomsky hierarchy and computation

Grammar	Languages	Automaton
Type 3	Regular	Finite automata
Type 2	Context-free	Pushdown automata (finite + stack)
Type 1	Context-sensitive	Bounded Turing machines
Type 0	Recursively enumerable	Turing machines

Reading

This week

- *Programming Languages Concepts*, Chapter 3
- Mogensen's *Introduction to Compiler Design*, Sections 1.1–1.8 and 2.1–2.5
- Tutorial on regular expressions and automata from course website

Next week

- *Programming Languages Concepts*, Chapter 4
- Mogensen's *Introduction to Compiler Design*, Sections 2.11, 2.12, 2.16

Exercises

- Exercises for week 35 due Wednesday:
  1.1, 1.2, 1.4, 2.1, 2.2, 2.3, and optionally 2.6

- Exercises for week 36 due 11 September:
  3.2, 3.3, 3.4, 3.5, 3.6, 3.7