SODA presentation, January 10th 2000

Faster deterministic dictionaries

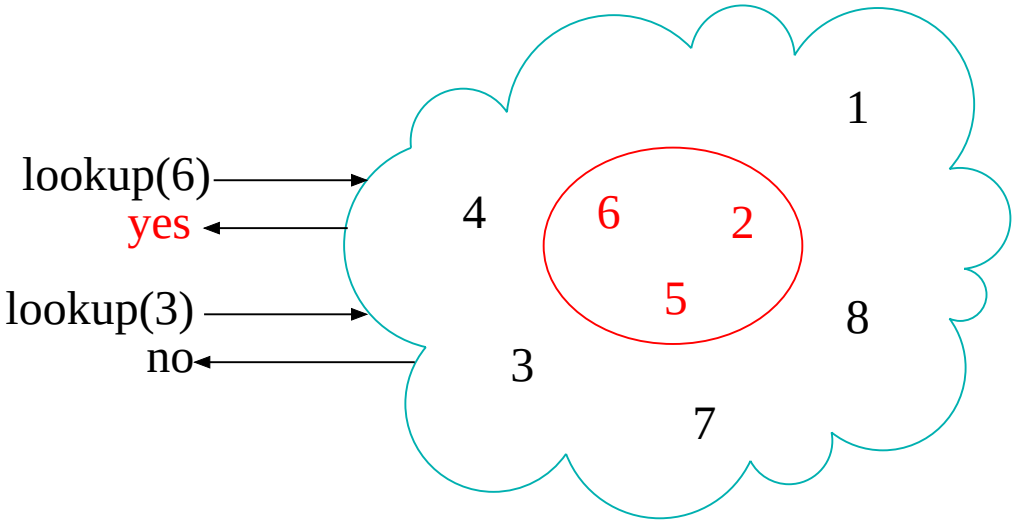
Rasmus Pagh

BRICS

Aarhus University, Denmark

Dictionaries

A dictionary stores a set S of n elements from a finite universe U.



[Updates: Insertion and deletion of elements.]

Performance issues

Model of computation:

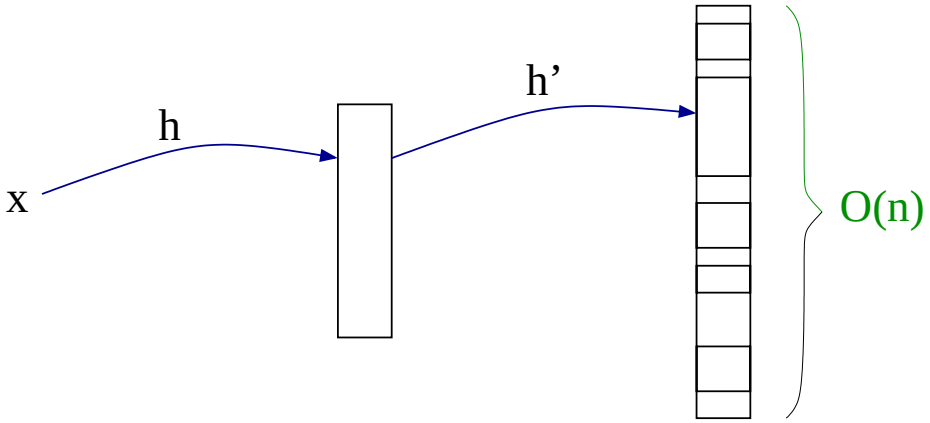
- Unit cost RAM with word size w and a standard instruction set, including multiplication.
- We allow compile-time computation (weak non-uniformity).
- $U = \{0, 1\}^w$.

We consider the asymptotics of:

- Lookup time.
- Space usage.
- Construction time/update time.

. . . as functions of n [and w].

Randomized dictionaries



Fredman, Komlós and Szemerédi (1982) designed a static dictionary with:

Lookup time: Worst case $O(1)$.

Space usage: $O(n)$ machine words.

Construction time: Expected $O(n)$ by a randomized algorithm.

Dietzfelbinger et al. (1988): Updates in $O(1)$ expected amortized time.

Removing randomness

Theme of this talk:

Using $O(n)$ words of space, how well can we do without random bits?

Motivation:

- Theoretical understanding.
- Random bits may not be available.
- Random bits may be expensive.
- No guarantee against bad performance.

We consider first the static case, then (briefly) the dynamic case.

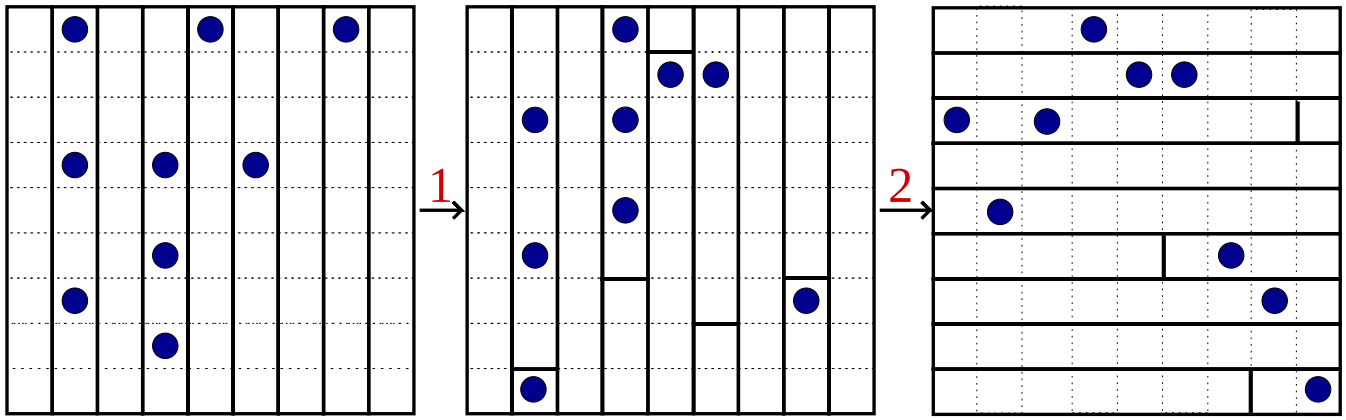
Some deterministic dictionaries

Reference	Lookup time	Construction time
Alon-Naor '94	$O(w/\log n)$	$O(nw\log^4 n)$
Andersson '96	$O(\log w \ \log\log n)$	$O(n)$
Miltersen '98	$O(1)$	$O(n^{1+\epsilon})$
Hagerup '99	$O(\log\log n)$	$O(n\log n)$
This talk	$O(1)$	$O(n\log n)$

[Dictionaries with lookup time $(\log n)^{\Omega(1)}$ not included]

Double displacement

Tarjan and Yao (1979) defined a class of hash functions for $|U| = O(n^2)$.



Look at U as an $O(n) \times O(n)$ grid. The "double displacement" process is:

1. Displace each column (circularly), and then
2. Displace each row (circularly).

The final rows define the hash function.

Fact: Displacement values can always be chosen such that this function is 1-1 on S.

Algorithms for finding displacement values

Tarjan and Yao's algorithm:

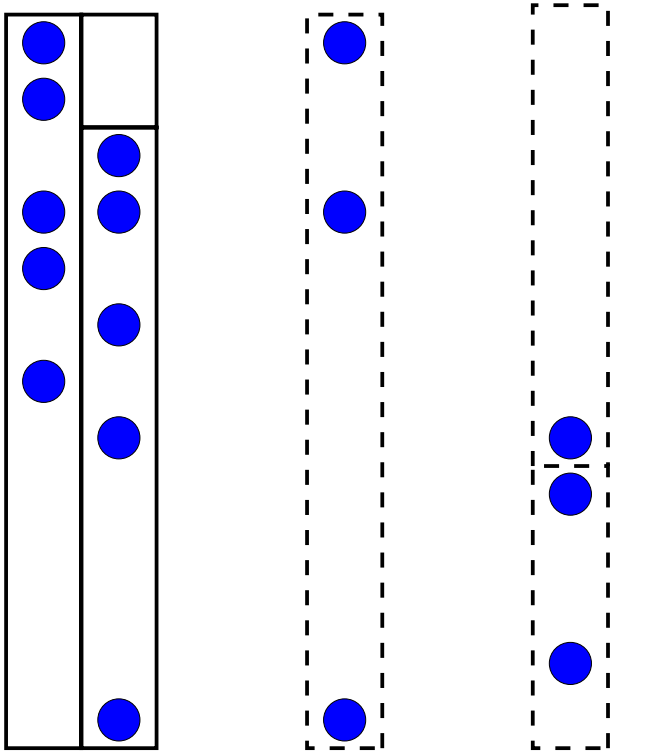
First-Fit-Decreasing search: $O(n^2)$ time, deterministically.

Our two-step approach: First randomize, then derandomize

1. Random-Decreasing search: $O(n)$ time using $O(n \log n)$ random bits (exp.)

2. Derandomized-Random-Decreasing search: $O(n \log n)$ deterministic time.

1. What is a “good” displacement value?



— one that does not result in (many) more collisions than what could be expected by a random choice

2. Derandomization

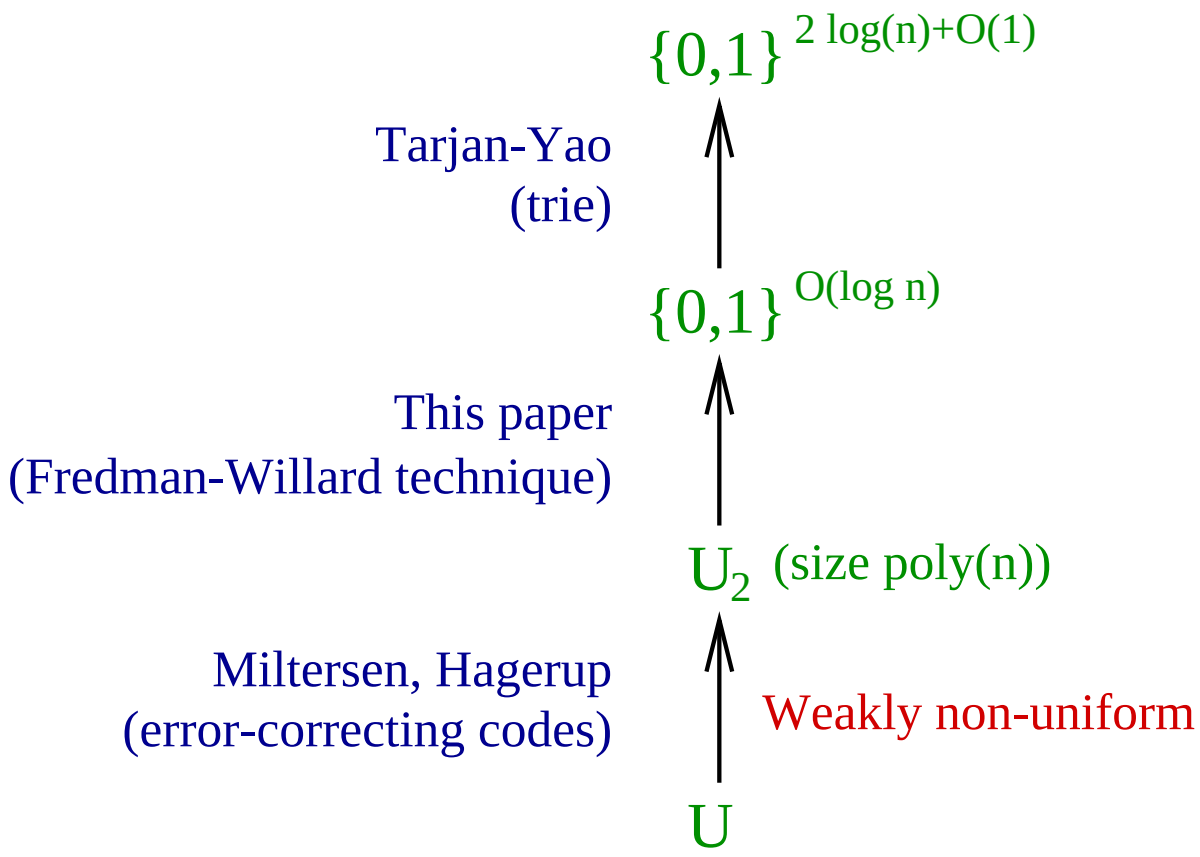
The essential facts:

- We can (tentatively) fix bits of a displacement value and efficiently compute the *conditionally* expected number of collisions when choosing the remaining bits at random.
- This is particularly simple if we exchange the cyclic shift with a permutation based on bit-wise exclusive or.

Theorem 1 *For $|U| = O(n^2)$ there is a static dictionary occupying $O(n)$ words of space and with constant lookup time, which can be constructed deterministically in time $O(n \log n)$.*

Universe reduction

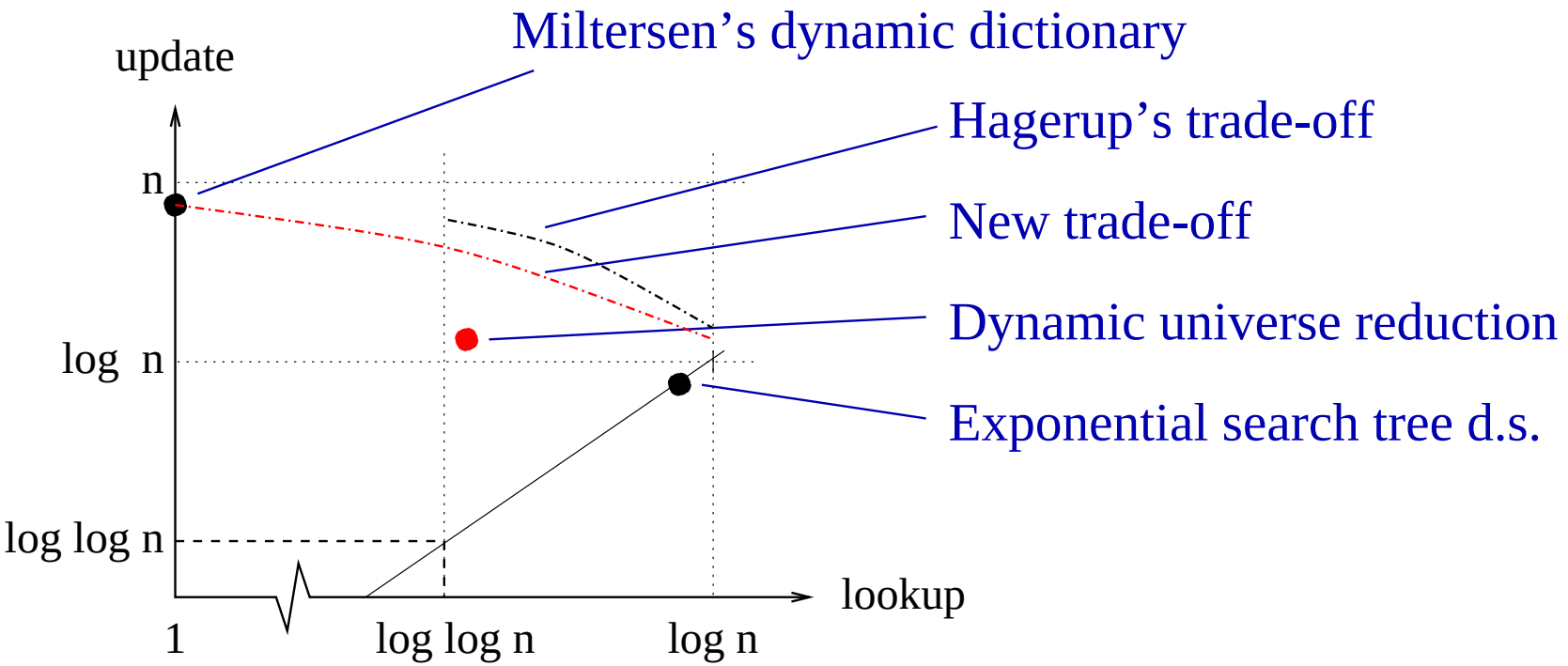
What remains is to reduce the general problem to the case $|U| = O(n^2)$.



Main theorem

Theorem 2 *There is a static dictionary occupying $O(n)$ words of space and with constant lookup time, which can be constructed deterministically in time $O(n \log n)$ by a weakly non-uniform algorithm.*

Removing randomness, dynamically



Some open problems

Static dictionaries:

Does randomness help at all?

Dynamic dictionaries:

Better deterministic upper/lower bounds.