CD F2007: Computation and Deduction
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Exercises 5

Every week we'll have lab excerises of which some need to be handed before class a week from the day they were assigned.

Today we did the proof of the Church Rosser theorem in class. In this homework your are asked to prove the same property for a calculus that consists only of pairs.

$$\text{Terms} \quad e \quad ::= \quad x \mid 0 \mid \langle e_1, e_2 \rangle \mid \text{let } \langle x, y \rangle = e_1 \text{ in } e_2$$

The judgment of parallel reduction for pairs and lets is written as $e \Longrightarrow e'$ and defined by the following three rules.

$$\frac{}{0 \Longrightarrow 0}\ \text{p\_0} \qquad \frac{e_1 \Longrightarrow e_1' \quad e_2 \Longrightarrow e_2'}{\langle e_1, e_2 \rangle \Longrightarrow \langle e_1', e_2' \rangle}\ \text{p\_pair}$$

$$\frac{e_1 \Longrightarrow e_1' \qquad \begin{array}{c} \overline{x \Longrightarrow x}\ u \quad \overline{y \Longrightarrow y}\ v \\ \vdots \\ e_2 \Longrightarrow e_2' \end{array}}{\text{let } \langle x, y \rangle = e_1 \text{ in } e_2 \Longrightarrow \text{let } \langle x, y \rangle = e_1' \text{ in } e_2'}\ \text{p\_let}^{x,u,y,v}$$

$$\frac{e_1 \Longrightarrow e_1' \quad e_2 \Longrightarrow e_2' \qquad \begin{array}{c} \overline{x \Longrightarrow x}\ u \quad \overline{y \Longrightarrow y}\ v \\ \vdots \\ e_3 \Longrightarrow e_3' \end{array}}{\text{let } \langle x, y \rangle = \langle e_1, e_2 \rangle \text{ in } e_3 \Longrightarrow [e_1'/x][e_2'/y]e_3'}\ \text{p\_beta}^{x,u,y,v}$$

We define the transitive closure as follows

$$\frac{}{e \Longrightarrow^* e}\ \text{refl} \qquad \frac{e_1 \Longrightarrow e_2 \quad e_2 \Longrightarrow^* e_3}{e_1 \Longrightarrow^* e_3}\ \text{trans}$$

Prove the following four theorems in Twelf.

Theorem 1 (Substitution)

$$\textit{If} \quad \begin{array}{c} \overline{x \Longrightarrow x}\ u \quad \overline{y \Longrightarrow y}\ v \\ \vdots \\ e \Longrightarrow e' \end{array} \quad \textit{then} \quad \begin{array}{c} \overline{m \Longrightarrow m'}\ u \quad \overline{n \Longrightarrow n'}\ v \\ \vdots \\ e[m/x][n/y] \Longrightarrow e'[m'/x][n'/y]. \end{array}$$

Theorem 2 (Diamond)
If $e \Longrightarrow e'$ *and* $e \Longrightarrow e''$ *then there exists a* e'''*, s.t.* $e' \Longrightarrow e'''$ *and* $e'' \Longrightarrow e'''$*.*