CPSC 629: Deductive Systems

Assignment 1

Due: 9/21/2000, 4pm

1. Show that the function *double* is well-typed.

$$double = \mathbf{fix}\ f.\,\mathbf{lam}\ x.\,\mathbf{case}\ x\ \mathbf{of}\ \mathbf{z}\ \Rightarrow \mathbf{z} \mid \mathbf{s}\ x' \Rightarrow \mathbf{s}\ (\mathbf{s}\ (f x'))$$

2. Consider an extension of the language by the unit type 1 and disjoint sums $\tau_1 + \tau_2$:

$$\begin{array}{llll} \textit{Types}: & \tau & ::= & \ldots \mid 1 \mid (\tau_1 + \tau_2) \\ \textit{Expressions}: & e & ::= & \ldots \mid \langle\rangle \mid \mathbf{inl}\ e \mid \mathbf{inr}\ e \mid (\mathbf{case}\ e_1\ \mathbf{of}\ \mathbf{inl}\ x_2 \Rightarrow e_1 \mid \mathbf{inr}\ x_3 \Rightarrow e_3) \end{array}$$

   (a) Give appropriate rules for evaluation and typing.

   (b) Extend the notion of value.

   (c) Extend the proof of value soundness from class (Theorem 2.1 in course notes).

   (d) Extend the proof of type preservation from class (Theorem 2.6 in course notes).

3. Prove that v *Value* is derivable if and only if $v \hookrightarrow v$ is derivable.

4. Prove the Replacement Lemma:

   If, for any type τ', $\vdash e_1' : \tau'$ implies $\vdash e_2' : \tau'$, then $\vdash [e_1'/x]e : \tau$ implies $\vdash [e_2'/x]e : \tau$.