## CPSC 629: Deductive Systems

# Assignment 3

### Due: 10/17/2000, 4pm

1. Lists of natural numbers are represented as follows in Twelf:

```
nat : type.
z : nat.
s : nat -> nat.

list : type.
nil : list.
; : nat -> list -> list.   %infix right 10 ;.
```

   Give the definition the judgment "reverse" as a deductive system. "reverse $l_1$ $l_2$" should be derivable whenever $l_2$ is the reverse list of $l_1$, and argue for the adequacy of the encoding.

2. Prove the adequacy of the representation of the judgment "an expression is closed" (soundness and completeness direction) for the **app** , **lam** , **fix** , **let** fragment of Mini-ML. Hint: The **lam** case has been shown in class.

3. Write Twelf programs to determine if a Mini-ML expression is free of the recursion operation **fix** and at the same time

   (a) linear (every bound variable occurs exactly once)

   (b) affine (every bound variable occurs at most once)

   (c) relevant (every bound variable occurs at least once)

   Since only one branch in a case statement will be taken during evaluation, a bound variable must occur exactly once in each branch in a linear expression, may occur at most once in each branch in an affine expression, and must occur at least once in each branch in a relevant expression.