Discrete Mathematics
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Homework 1

due Tuesday, 11.09. 2012
before class meets.

In this problem, we study a different flippling operation on strings (or also called configurations) of blue and white pebbles: Just like in class, we traverse the string of pebbles from left to right. We define the recipe as follows:

Definition: *We apply the following rule until one or no pebbles are to be considered, in which case we return the remainder. The rule is given as follows: if we encounter*

- *white followed by white, we turn both into blue pebbles and skip the next,*
- *blue followed by white, we turn both into white pebbles and skip the next,*
- *white followed by blue, we flip them to blue followed by white and skip,*
- *blue followed by blue, we flip the first to white, keep the second to be blue, and skip the next.*

Problem 1

Start with a configuration of eight pebbles, four white pebbles followed by four blue pebbles. Perform the aforementioned operation until you have reached a cycle.

Problem 2

Write out an automaton, that visualizes the aforementioned algorithm.

Problem 3

Given a string of n pebbles of any form, conjecture after how many rounds you will reach a cycle. Formulate your conjecture in the following way.

Conjecture: *Let n be the length and s an inital configuration of pebbles s. We need to apply the recipe at least ____ times, to arrive at the configuration s again.*

Problem 4 [Extra Credit]

Argue informally, why your conjecture from Problem 3 is true.