Discrete Mathematics
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Homework 11

due *before* class meets.

1. What is the probability that when a coin is flipped six times in a row , t lands heads up every time?
2. What is the probability that a five-card poker hand contains exactly one ace?
3. What is the probability that a five-card poker hand contains a straight flush, that is, the 10, jack, queen,king, and ace of one suit?
4. What is the probability that Bo Colleen, Jeff, and Rohini win the first, second, and third prices, respectively, in a drawing if 200 people enter a contest and
   (a) no one can win more than one prize.
   (b) winning more than one prize is allowed.
5. Which is more likely; rolling a total of 8 when two dice are rolled or rolling a total of 9 when tree dice a rolled?
6. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.
   (a) E_1: the first coin comes up tails; E_2: the second coin comes up heads.
   (b) E_1: the first coin comes up tails; E_2: two, and not three, heads come up in a row.
   (c) E_1: the second coin comes up tails; E_2: two, and not three, heads come up in a row.
7. A pair of cie is loaded. The probability that a 4 appears on the fist die is $2/7$ and the probability that a 3 appears on the second die is $2/7$. Other outcomes for each die appear with probability $1/7$. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled?
8. Show that if E and F are events, then $p(E \cap F) \geq P(E) + P(F) - 1$ This is known as Bonferroni's Inequality.
9. Use mathematical induction to prove the following generalization of Bonferroni's Inequality:

$$p(E_1 \cap E_2 \ldots \cap E_n) \leq \sum_{i=1}^{n} p(E_i) - (n-1)$$

where E_i are n events.