

Homework 12

due *before* class meets.

1. Suppose that E and F are events in a sample space and $p(E) = 1/3$, $p(F) = 1/2$, and $p(E \mid F) = 2/5$. Find $p(F \mid E)$.
2. Suppose that one person in 100000 has a particular rare disease for which there is a fairly accurate diagnostic test. This test is correct 99% of the time when given to someone with the disease; it is correct 99.5% of the time when given to someone who does not have the disease. Given this information, can we find
 - (a) the probability that someone who tests positive for the disease has the disease?
 - (b) the probability that someone who tests negative for the disease does not have the disease?

Should someone who tests positive be very concerned that she or he has the disease?

3. Suppose that Ann selects a ball by first picking one of two boxes at random and then selecting a ball from this box. The first box contains three orange balls and four black balls, and the second box contains five orange balls and six black balls. What is the probability that Ann picked a ball from the second box if she has selected an orange ball?
4. Suppose that E is an event from a sample space S and that F_1, F_2, \dots, F_n are mutually exclusive events such that

$$\bigcup_{i=1}^n F_i = S.$$

Assume that $p(E) \neq 0$ and $p(F_i) \neq 0$ for $i = 1, 2, \dots, n$. Show that

$$p(F_j \mid E) = \frac{p(E \mid F_j)p(F_j)}{\sum_{i=1}^n p(E \mid F_i)p(F_i)}$$

Hint: Use the fact that $E = \bigcup_{i=1}^n (E \cap F_i)$.