Exercises for Lecture 3

1. Recall the definition of logarithms. We define the logarithm n to base b of x, written as $n = \log_b x$ if and only if $b^n = x$. Prove the following theorems.

   (a) Show that $\log_b(xy) = \log_b(x) + \log_b(y)$.

   (b) Show that $\log_b(x^n) = n \log_b(x)$.

   (c) Show that $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$. [Hand-in]

   (d) Show that $\log_b(a) = \frac{1}{\log_a(b)}$.

   (e) For all natural numbers n and bases b, $\log_b n < n$. [Hand-in]

2. Recall trigonometry from your school, including the definition of the trigonometric function $\sin(x)$ and $\cos(x)$. You might remember that the following laws hold:

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \tag{1}$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \tag{2}$$

   Prove the following by induction on n the following: For all j and for all n

$$(\cos(x) + j\sin(x))^n = \cos(nx) + j\sin(nx)$$