Discrete Mathematics
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Homework 4

due *before* class meets.

1. Let W be the set of all women in the world. Express the following formulas in natural language:
   (a) $\forall w \in W.w$ likes chocolate
   (b) $\exists w \in W.w$ is a mother
   (c) $\neg\exists w W.w$ is a father
2. Determine whether the following claims are true or not (no proofs!), and rewrite the claims as formulas containing quantifiers.
   (a) every natural number is positive
   (b) every integer is smaller than (or equal to) 0 or greater than 0
   (c) all integers are prime numbers
   (d) there exists a natural number which is greater than 7
   (e) there is no natural number which is smaller than 7 (f) every integer is a prime number
   (f) if x is an integer, then x2 is greater than or equal to 0
3. Prove the following theorems formally using the rules of inference from class. We omit the domain associated with the quantifiers to signal, that these theorem holds independent of the choice of domain.
   (a) $\forall x.\, P(x,x) \Rightarrow \forall x.\, \exists y.\, P(x,y)$.
   (b) $(\forall x.\, \neg P(x)) \Rightarrow \neg(\exists x.\, P(x))$
   (c) $\forall x.\, (P(x) \Rightarrow \neg\forall y.\, (\neg P(y)))$
   (d) $(\forall x.\, P(x)) \Rightarrow \neg\exists y.\, \neg P(y)$
   (e) $((\exists x.\, P(x)) \Rightarrow (\forall y.\, Q(y))) \Rightarrow \forall z.(P(z) \Rightarrow Q(z))$.