Discrete Mathematics
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Homework 5

due *before* class meets.

1. For each of the following expression in the Java language, give an equivalent λ-term:
   (a) Example: `int addfive (int x) { return x+5; }`
   Solution: $\lambda x : x + 5$
   (b) Example: `int add (int x, int y) { return x+y }`
   Solution: $\lambda x : \lambda y : x + y$
   (c) Example: `double sinpone (double x) { return Math.sin(x+1); }`
   Solution: $\lambda x : sin(x + 1)$
   (d) `int seven (int x) { return 7; }`
   (e) `int fst (int x, int y) { return x; }`
   (f) `int snd (int x, int y) { return y; }`
   (g) `int dostuff (int x) { return (x+5)/9; }`
   (h) `int sumprices (hat h, jacket j) { return price(h)+price(j); }`
   (i) `Object apptoitself (function f) { return f(f); }`
   (j) `Object apptwice (function f, int x) { return f(f(x)); }`
2. Reduce the following λ-terms one step at a time. Warning: not all of the evaluate to a number.
   (a) Example: $((\lambda x.\, \lambda y.\, xy)(\lambda z.\, z^3))\ 6$ Solution:

$$\begin{aligned} ((\lambda x.\, \lambda y.\, xy)(\lambda z.\, z^3))\ 6 &= (\lambda y.\, (\lambda z.\, z^3)y)\ 6 \\ &= (\lambda z.\, z^3)\ 6 \\ &= 6^3 \\ &= 216 \end{aligned}$$

   (b) $((\lambda x.\, x)(\lambda y.\, y + y))\ 7$
   (c) $((\lambda x.\, \lambda y.\, x + y)((\lambda z.\, 14)\ 5))((\lambda u.\, u - 4)\ 6)$
   (d) $(((\lambda x.\, \lambda y.\, \lambda z.\, x\ (y\ (x\ z)))(\lambda u.\, u + 4))(\lambda v.\, v^2))\ 8$
   (e) $((\lambda x.\, \lambda y.\, x \times (y - 21))(\lambda z.\, z \times 4))\ 6$
3. (a) Define a λ-expression e that composes two functions f and g in the following way: $e\ f\ g\ x = f(g(x))$
   (b) Give a reduction of $e\ (\lambda x.\, 2 \times x)\ (\lambda y.\, 3 \times y)$
   (c) How many steps did it take to reduce this expression?
   (d) Is the reduction sequence unique?