Discrete Mathematics
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Homework 6

due *before* class meets.

1. **Sets**
   Let $A = \{1, -2\}$, $B = \{1, -1, 2\}$ and $C = \{1, \{2, 3\}\}$.
   (a) What is the the power-set of B? And the power-set of C?
   (b) Which of the following statements is true? Argue why!
      i. $\{1\} \subset A$
      ii. $1 \in C$
      iii. $\{1\} \in A$
      iv. $1 \subset A$
      v. $3 \in C$
      vi. $1 \in A$
      vii. $\{1\} \in C$
      viii. $A \in B$
      ix. $\{2, 3\} \in C$
      x. $B \subset A$
      xi. $\{2\} \in C$
2. **Powerset**
   Let $A = \{-1, 0, 1\}$ and $B = \{1, 2\}$. Write explicitly the sets $A \times B$, $A \times A$, $(A \times A) \cap (A \times B)$, $A \times (A \cap B)$, $(A \times A) \cup (A \times B)$, $\mathcal{P}(B \times B)$ and $\mathcal{P}(B) \times \mathcal{P}(B)$.
3. **Relations:** In year 2052, management has decided to award an iPhone 50G-Teleport to the best teacher and the best student at ITU. In order to choose the winners, all students must vote for two teachers and all teachers must express a single vote for their favourite student. Given the set P of people at ITU (consisting of both students and teachers), the relation $V \subset P \times P$ expresses whether some person p_1 has voted for some other person p_2. Moreover, the relation M relates two persons whenever they have voted for each other. Discuss the properties (reflexive, symmetric, antisymmetric, transitive) of relations V and M.
4. **Quotient Sets:** Given a set A and a binary relation $R \subset A \times A$, such that R is reflexive, transitive and symmetric, we define a quotient set Q as the set of all subsets that are considered equal under the relation R: Let $[a] = \{x \in A | aRx\}$ and $Q = \{[a] | a \in A\}$. *In these situations, we often write $Q = A|_R$, and call Q the quotient of A by R.*
   (a) If A is the set of all cars, give examples of R and Q.
   (b) If A is the set of the natural numbers and $Q = \{0, 1, \ldots n\}$ give a justifiable definition of R.