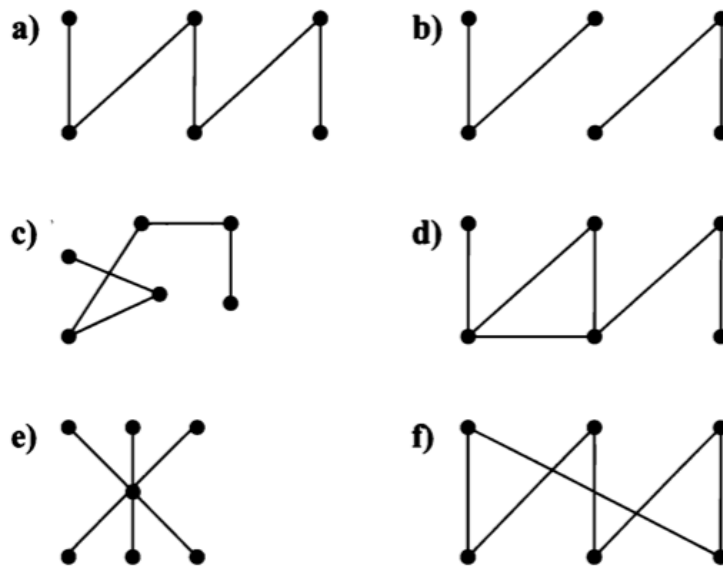
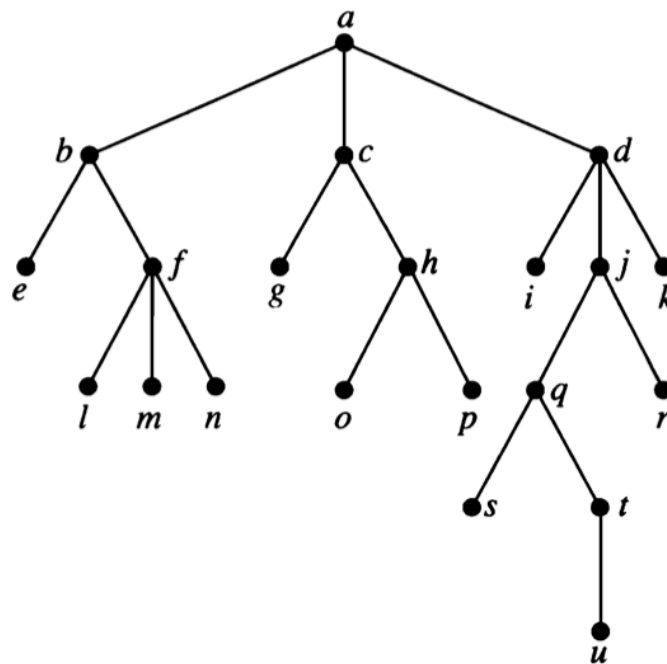


Exercises for Lecture 7

1) Which of these graphs are trees?



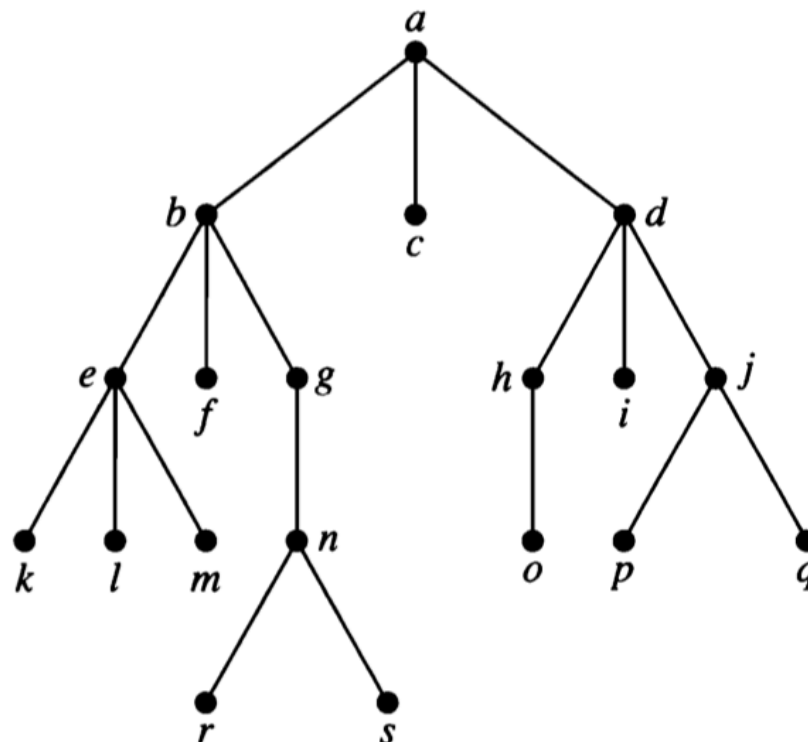
2) Answer the questions about the rooted tree illustrated



Questions:

- Which vertex is the root?
- Which vertices are internal?
- Which vertices are leaves?
- Which vertices are children of *j*?
- Which vertex is the parent of *h*?
- Which vertices are siblings of *o*?
- Which vertices are ancestors of *m*?
- Which vertices are descendants of *b*?

3) Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.



4) What is the value of each of these prefix expressions?

- $- * 2 / 8 4 3$
- $\uparrow - * 3 3 * 4 2 5$
- $+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$
- $* + 3 + 3 \uparrow 3 + 3 3 3$
- $+ - * 2 3 5 / \uparrow 2 3 4$

Hint: Reconstruct a tree from the expression, and then compute the value of a tree by evaluating the subtrees and then compute the value as defined by the mathematical operator. \uparrow stands for exponentiation.

5) Follow these steps to give a recursive definition of the factorial function

$F(n)=n!$

- Specify the initial value of this function
- Give a rule for finding $F(n + 1)$ from $F(n)$
- Determine a value of the factorial function, such as $F(5) = 5!$

6) Suppose that f is defined recursively by

$$f(0) = 5$$

$$f(n+1)=4f(n)+5$$

Find $f(n)$ for $n=1\dots 5$ that is $f(1), f(2), f(3), f(4), f(5)$

7) We define the length of a list l as

$$\text{length}(l) = 0$$

$$\text{if } l = \text{nil}$$

$\text{length}(l) = 1 + \text{length}(l')$ if $l = x::l'$

Prove by structural induction that:

$\text{length}(l_1 @ l_2) = \text{length}(l_1) + \text{length}(l_2)$

8) Solve exercise 3 from the course notes.

9) [Extra credit] Solve exercise 4 from the course notes.