

Exercises for Lecture 2: Logic

1. Write down all the rules and axioms presented during the lecture.
2. Let A be the proposition “Anne is Danish” and B the proposition “Bart is American”. Write down the formulae formalizing the following claims.
 - (a) Anne is not Danish
 - (b) Anne is Danish if Bart is American
 - (c) Anne is Danish and Bart is American
 - (d) If Bart is American, then Bart is not American
 - (e) It is not true that Anne is Danish and Bart is American
 - (f) If Anne is not Danish, then Bart is not American
 - (g) Anne is Danish if and only if Bart is American
 - (h) Either Anne is Danish or Anne is not Danish
 - (i) If Anne is Danish and Bart is American, then Bart is American
 - (j) Either Anne is Danish, or, if Anne is not Danish, then Bart is American.
 - (k) Either Anne is Danish and Bart is American, or Anne is Danish and Bart is not American.
3. Let A be the proposition “Anja comes to the party” and B the proposition “Beatrix come to the party” and C the proposition “Charlotte comes to the party”. Write down the formulae formalizing the following claims.
 - (a) If Beatrix comes to the party then Anja and Charlotte do.
 - (b) Anja comes to the party if Beatrix and Charlotte don’t
 - (c) Beatrix comes to the party, while Anja doesn’t
 - (d) Charlotte doesn’t come to the party unless Beatrix does
 - (e) Either Charlotte comes to the party, or Anja and Beatrix are not coming
 - (f) Anja comes to the party if Charlotte doesn’t come unless Beatrix does

- (g) If Beatrix comes to the party then Anja does, but if Beatrix doesn't come Charlotte does
 - (h) Charlotte and Anja come to the party if Beatrix doesn't, but if Charlotte comes Anja does not
 - (i) If Charlotte comes to the party, then Beatrix comes if Anja does.
 - (j) If Beatrix comes to the party, then if Anja doesn't Charlotte does.
 - (k) Anja comes to the party if Charlotte does, but if Charlotte does, Beatrix doesn't
4. Prove or disprove the following statements. State clearly if you used the rule of the excluded middle or not.

- (a) $\overline{A \vee \neg A}$
- (b) $\frac{A \wedge B}{\neg(A \rightarrow \neg B)}$
- (c) $\frac{A \wedge B}{\neg(\neg A \vee \neg B)}$
- (d) $\frac{A \rightarrow B \quad A \rightarrow \neg B}{\neg A}$
- (e) $\frac{A \rightarrow B \quad A \rightarrow C}{A \rightarrow (B \wedge C)}$
- (f) $\frac{A \rightarrow \neg B \quad A \rightarrow \neg C}{\neg(A \wedge (B \vee C))}$
- (g) $\frac{A \wedge B \quad \neg C \rightarrow \neg B}{C}$
- (h) $\frac{A \vee B \quad C \rightarrow \neg B}{C \rightarrow A}$

5. Prove the following formulae.

- (a) $(A \wedge B) \rightarrow A$
- (b) $(A \vee B) \rightarrow A$
- (c) $(A \vee B) \rightarrow (A \wedge B)$
- (d) $(A \wedge B) \rightarrow (A \vee B)$

- (e) $((A \rightarrow B) \rightarrow B) \rightarrow B$
- (f) $(A \wedge B) \vee (\neg A \vee \neg B)$
- (g) $(A \wedge B) \wedge C \rightarrow (A \wedge B)$
- (h) $(A \vee B) \vee C \rightarrow (A \vee B)$
- (i) $((A \rightarrow B) \rightarrow B) \vee \neg B$
- (j) $\neg((A \rightarrow \neg B) \wedge B)$
- (k) $\neg((A \wedge B) \rightarrow C) \rightarrow (B \wedge \neg(A \wedge B))$

6. Let $P \equiv Q$ mean $(P \rightarrow Q) \wedge (Q \rightarrow P)$. Prove the following formulae.

- (a) $A \rightarrow B \equiv \neg(A \wedge \neg B)$
- (b) $A \rightarrow B \equiv (\neg A \vee B)$
- (c) $(A \rightarrow B) \wedge (B \rightarrow C) \equiv A \rightarrow C$
- (d) $(\neg A \rightarrow B) \vee (\neg A \vee B) \equiv B$
- (e) $A \wedge \neg(A \wedge \neg B) \equiv C \rightarrow \neg(\neg B \wedge \neg C)$