Exercises for Lecture 9: λ-calculus

1. For each of the following expression in the Java language, give an equivalent λ-term:

   (a) Example: int addfive (int x) { return x+5; }
   Solution: $\lambda x.\ x + 5$

   (b) Example: int add (int x, int y) { return x+y }
   Solution: $\lambda x.\lambda y.\ x + y$

   (c) Example: double sinpone (double x) { return Math.sin(x+1); }
   Solution: $\lambda x.\ sin(x + 1)$

   (d) int seven (int x) { return 7; }

   (e) int fst (int x, int y) { return x; }

   (f) int snd (int x, int y) { return y; }

   (g) int dostuff (int x) { return (x+5)/9; }

   (h) int sumprices (hat h, jacket j) { return price(h)+price(j); }

   (i) Object apptoitself (function f) { return f(f); }

   (j) Object apptwice (function f, int x) { return f(f(x)); }

2. Reduce the following λ-terms one step at a time. Warning: not all of the evaluate to a number.

   (a) Example: $((\lambda x.\lambda y.\ xy)\ (\lambda z.\ z^3))\ 6$
   Solution:

$$\begin{aligned} & ((\lambda x.\lambda y.\ xy)\ (\lambda z.\ z^3))\ 6 \\ =&(\lambda y.\ (\lambda z.\ z^3)y)\ 6 \\ =&(\lambda z.\ z^3)\ 6 \\ =&6^3 \\ =&216 \end{aligned}$$

   (b) $((\lambda x.\ x)(\lambda y.\ y + y))\ 7$

   (c) $((\lambda x.\lambda y.\ x + y)((\lambda z.\ 14)\ 5))((\lambda u.\ u - 4)\ 6)$

   (d) $(((\lambda x.\lambda y.\lambda z.\ x(y(xz)))(\lambda u.\ u + 4))(\lambda v.\ v^2))\ 8$

   (e) $((\lambda x.\lambda y.\ x * (y - 21))(\lambda z.\ z * 4))\ 6$

(f) $((\lambda x.\lambda y.\ xy)(\lambda z.\ zz))(\lambda u.u)$

(g) $(((\lambda x.\lambda y.\ xy)(\lambda z.\lambda u.\ z(uz)))(\lambda v.v))\ 5$