## Sample Exam like Questions

## November 8, 2011

- Give a derivation of the statement (A ⇒ B) ⇒ ((A ⇒ ¬B) ⇒ ¬A).
  Give a derivation of ∃a ∈ 𝔅. ∃b ∈ 𝔅. ∃c ∈ 𝔅. a<sup>2</sup> + b<sup>2</sup> = c<sup>2</sup>.
- 2. Let  $R = a \times b \times (a \times b)^*$  be a regular expression. Define a finite automaton that computes the language of R.
  - Show that the language of  $R \times S$  and the language of  $R \times (R \times 0 + S \times 1)$  result in the same set.
- 3. Define the length of a list as a recursive function.
  - Let L and K be two lists of natural numbers. Prove that  $length(L_1 @ L_2) = length(L_1) + length(L_2)$ .
  - Give the trace of computing length([1,2]@[3,4]). How many steps does it take?
- 4. Consider a perfect binary tree with  $2^n$  leaves. Prove that the path from the root to the leaf is always n.
  - How many nodes are in a perfect binary tree of hight n? Come up with a conjecture and prove it.