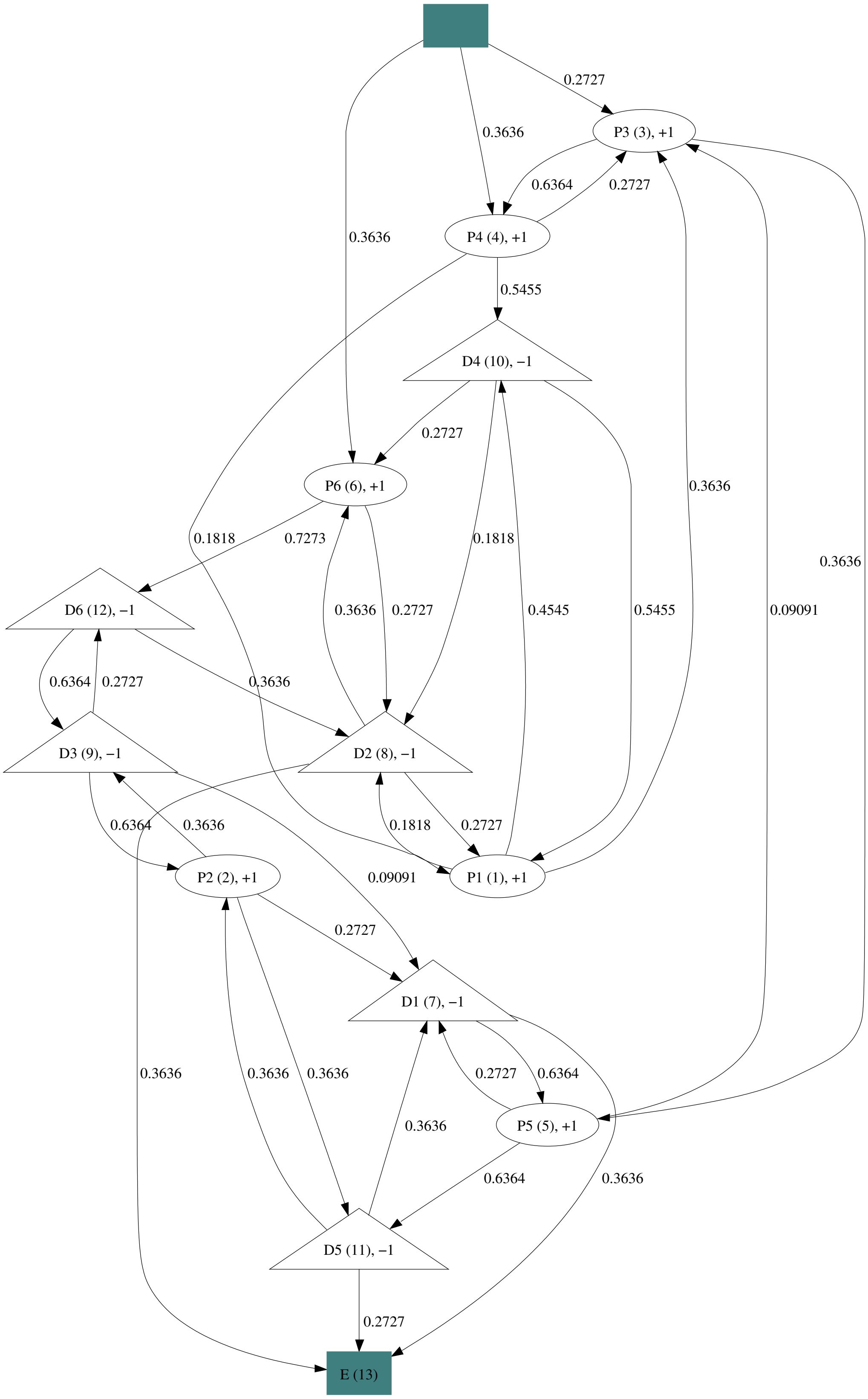
The dial a ride problem (DARP)



The dial a ride problem (DARP)

Work in progress! Most of the material presented in this talk is from:

Jean-François Cordeau, *A Branch-and-Cut Algorithm for the Dial-a-Ride Problem*, technical report CRT-2004-23.

Some material is new and is based on joint work with Jean-François Cordeau and Gilbert Laporte (Canada Research Chair in Distribution Management, HEC Montréal).

About myself

- Stefan Røpke (sropke@diku.dk)
- PhD student at DIKU
- Interested in solving routing problems using metaheuristics and exact optimization methods.

The dial a ride problem

The problem

- Door-to-door transportation of elderly and disabled persons (*the users*).

- Several users are transported in the same vehicle (think of a mini-bus).

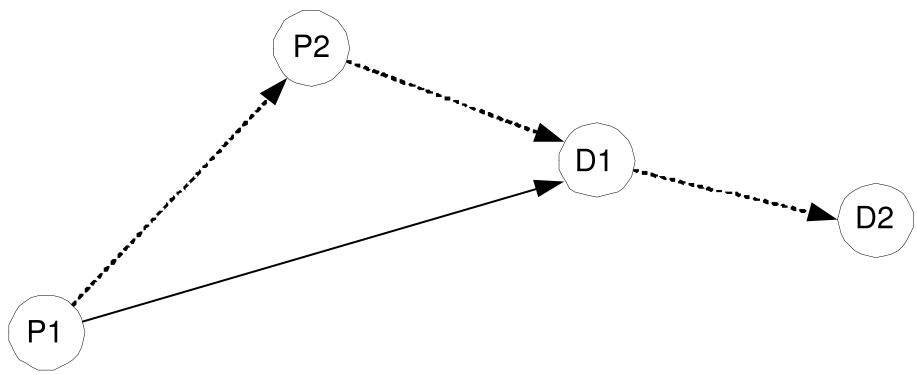
- The users specify when they wish to be picked up and when the they have to be at their destination. Such a transportation task is denoted a *request.*

- The users do not specify an exact time of day, but a time window. Example: Instead of requesting a pickup at 9:11 the users request a pickup between 9:00 and 9:30.

- Often the user only specify either the pickup or delivery time window. An operator would assign the other time window.

The dial a ride problem

- Users don't like to be taken on long detours even if it helps the overall performance of the transportation system. Consequently a maximum ride time constraint is specified for each request.



- Time windows are not enough for ensuring that the maximum ride time constraint is enforced. Example: pickup [8:00; 8:15], delivery [8:45; 9:00], max ride time 45 minutes. Pickup at 8:00 and delivery at 9:00 violates max ride time constraint. Pickup time window could be shrunk to [8:15; 8:15]. This would ensure that ride time constraint is enforced, but it rules out perfectly good solutions like pickup at 8:05 and delivery at 8:45.

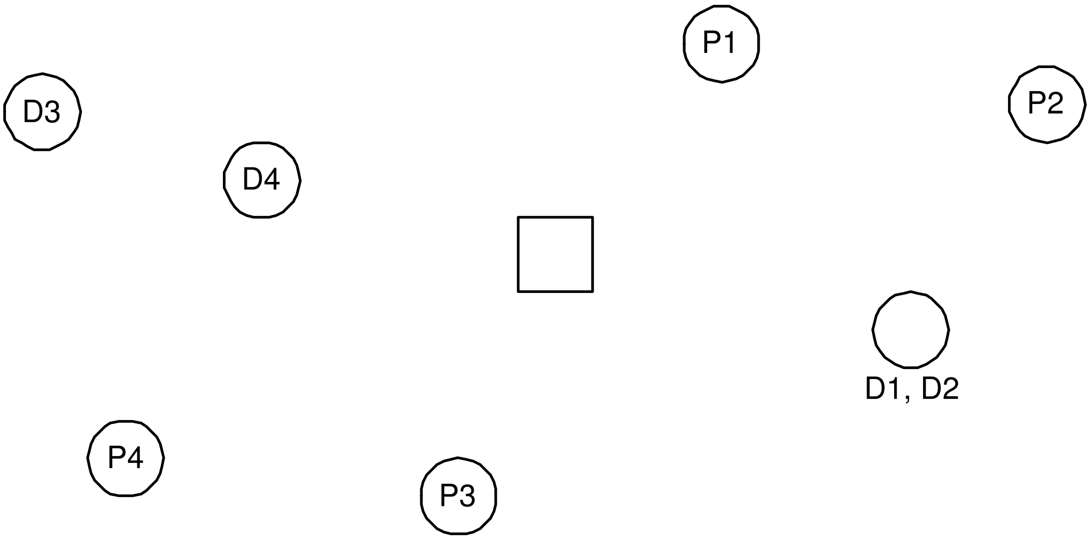
- Each vehicle has a certain capacity (only a limited amount of seats).

- The vehicles have to start and end their tours at a given start and end terminal.

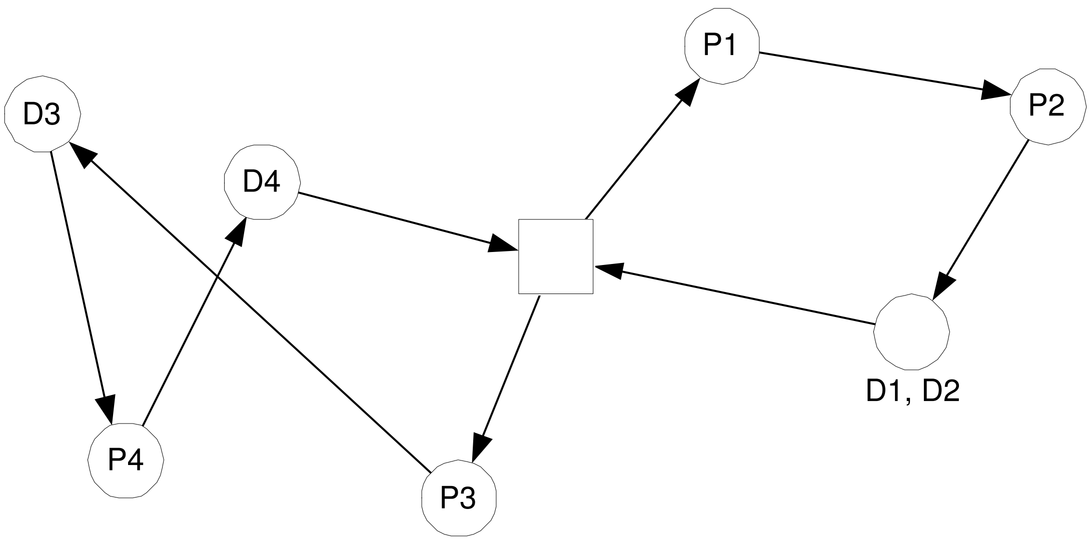
- Objective: minimize driving cost subject to the constraints mentioned above.

- Problem is NP-Hard.

DARP example



Possible solution:

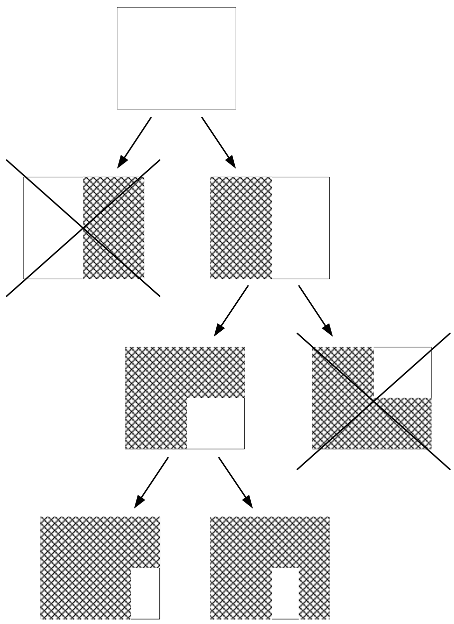


Branch and Bound

- Minimization problem. Main ingredients: lower and upper bound.

- High level algorithm:

  1. Set of subproblems (SoS) = { Entire problem }
  2. Remove subproblem S from SoS
  3. Find lower and upper bound (LB and UB) for S
  4. **if** $UB <$ global UB (GUB) **then** $GUB = UB$
  5. **if** $LB < GUB$ **then** split S into two subproblems and add them to SoS
  6. if $SoS \neq \emptyset$ **then goto** step 2, **else** return GUB



DARP formal definition (Graph problem)

Notation:

n	Number of requests.
$P = \{1, \ldots, n\}$	Pickup locations
$D = \{n+1, \ldots, 2n\}$	Delivery locations
$N = P \cup D \cup \{0, 2n+1\}$	The set of all nodes in the graph. 0 and $2n+1$ are the start and end terminal respectively. Request i consist of pickup i and delivery $n+i$.
K	Set of vehicles
$G = (N, A)$	Directed graph on which the problem is defined. A is the set of edges.
Q	Capacity of a vehicle
q_i	Amount loaded onto vehicle at node i. $q_i = q_{n+i}$.
$[e_i, l_i]$	time window of node i
$d_i > 0$	duration of service at node i
L	Max ride time of a request.
c_{ij}	Cost of traveling from node i to node j. It is Assumed that c_{ij} satisfies the triangle inequality.
t_{ij}	Time needed for going from node i to node j. It is assumed that t_{ij} satisfies the triangle inequality.

Standard model (DARP1)

Decision variables

Binary variables

x_{ij}^k 1 iff the kth vehicle goes straight from node i to node j.

Fractional variables

B_i^k When vehicle k starts visiting node i

Q_i^k The load of vehicle k after visiting node i.

L_i^k The ride time of request i on vehicle k.

Standard model (DARP1)

Objective:

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij}^k x_{ij}^k$$

Every request is served exactly once:

$$\sum_{k \in K} \sum_{j \in N} x_{ij}^k = 1 \quad \forall i \in P$$

Same vehicle services pickup and delivery:

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{n+i,j}^k = 0 \quad \forall i \in P, k \in K$$

Every vehicle leaves the start terminal:

$$\sum_{j \in N} x_{0j}^k = 1 \quad \forall k \in K$$

The same vehicle that enters a node leaves the node:

$$\sum_{j \in N} x_{ji}^k - \sum_{j \in N} x_{ij}^k = 0 \quad \forall i \in P \cup D, k \in K$$

Every vehicle enters the end terminal:

$$\sum_{i \in N} x_{i,2n+1}^k = 1 \quad \forall k \in K$$

Standard model (DARP1)

Setting and checking visit time:

$$B_j^k \geq (B_i^k + d_i + t_{ij}) x_{ij}^k \quad \forall i \in N, j \in N, k \in K$$

$$e_i \leq B_i^k \leq l_i \quad \forall i \in N, k \in K$$

Linearization of first equation (M_{ij}^k is a large constant):

$$B_j^k \geq B_i^k + d_i + t_{ij} - M_{ij}^k (1 - x_{ij}^k) \quad \forall i \in N, j \in N, k \in K$$

Setting and checking ride time:

$$L_i^k = B_{n+i}^k - (B_i^k + d_i) \quad \forall i \in P, k \in K$$

$$L_i^k \leq L \quad \forall i \in N, k \in K$$

Setting and checking vehicle load:

$$Q_j^k \geq (Q_i^k + q_j) x_{ij}^k \quad \forall i \in N, j \in N, k \in K$$

$$Q_i^k \leq Q \quad \forall i \in N, k \in K$$

Binary variables:

$$x_{ij}^k \in \{0, 1\} \quad \forall i \in N, j \in N, k \in K$$

Preprocessing

- Shrink time windows. For example if $l_i = 10, l_{n+i} = 20, d_i = 2$ and $t_{i,n+i} = 12$ then l_i can be reduced to 6.

- Remove edges from G that cannot be part of a feasible solution.

- Some examples

  – Edges that are impossible because of time windows
  – Edges of the type $(n+i, i) \forall i \in P$
  – Edges of the type $(0, n+i) \forall i \in P$ and $(i, 2n+1) \forall i \in P$
  – Edges that are impossible because of ride time constraints. Edge (i, j) can be removed if $j \neq n+i$ and the trip $i \rightarrow j \rightarrow n+i$ violates the ride time constraint of request i

- Preprocessing is fast and easy to do, but can have a significant impact on the running time of the algorithm.

Some results

- Solving (DARP1) using CPLEX 8.0 on 2.5 Ghz Pentium 4

Instance	Bound	CPU (min)	Nodes	Opt
a2-16	*294.25	0.01	23	294.25
a2-20	*344.83	0.05	313	344.83
a2-24	*431.12	1.42	10,868	431.12
a3-18	*300.48	0.41	3,596	300.48
a3-24	*344.83	76.59	310,667	344.83
a3-30	472.17	240.00	515,931	494.85
a3-36	570.26	240.00	504,553	583.19
a4-16	*282.68	21.49	145,680	282.68
a4-24	359.52	240.00	442,000	375.02
a4-32	427.65	240.00	189,900	485.50
a4-40	462.21	240.00	65,000	557.69
a4-48	466.7	240.00	40,400	668.82
b2-16	*309.41	0.21	5,815	309.41
b2-20	*332.64	0.01	26	332.64
b2-24	*444.71	2.76	32,399	444.71
b3-18	*301.64	1.29	12,223	301.65
b3-24	*394.51	7.27	42,950	394.51
b3-30	*531.44	189.74	574,281	531.45
b3-36	588.44	240.00	447,474	603.79
b4-16	*296.96	2.44	20,189	296.96
b4-24	*369.36	59.31	175,495	369.36
b4-32	460.78	240.00	222,600	494.82
b4-40	570.37	240.00	153,400	656.63
b4-48	577.64	240.00	50,000	673.81

Preprocessing pays off

- Some examples (Cplex 9.0 on 3.0Ghz Pentium 4).

	Preprocessing		No preprocessing	
	CPU (min)	Nodes	CPU (min)	Nodes
a2-20	0.07	332	0.31	1500
b3-24	3.91	16773	30.37	80161

- (DARP1) had $O(|N|^2|K|)$ binary variables. If we could get rid of the k index on the x_{ij}^k variables then the number of binary variables could be reduced to $O(|N|^2)$, which hopefully would make the problem easier to solve.

- (DARP2) - model where the k index is stripped from all variables. The variables have the same meaning as in (DARP1), they are just no longer associated with a specefic vehicle.

Compact model (DARP2)

Objective:

$$\min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

One vehicle enters every user node and one vehicle leaves every user node:

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in P$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall j \in P$$

Setting and checking visit time:

$$\begin{aligned} B_j \geq (B_i + d_i + t_{ij}) x_{ij} & \quad \forall i \in N, j \in N \\ e_i \leq B_i \leq l_i & \quad \forall i \in N \end{aligned}$$

Setting and checking ride time:

$$\begin{aligned} L_i = B_{n+i} - (B_i + d_i) & \quad \forall i \in P \\ L_i \leq L & \quad \forall i \in N \end{aligned}$$

Setting and checking vehicle load:

$$\begin{aligned} Q_j \geq (Q_i + q_j) x_{ij} & \quad \forall i \in N, j \in N \\ Q_i \leq Q & \quad \forall i \in N \end{aligned} \tag{1}$$

Binary variables:

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, j \in N$$

Compact model (DARP2)

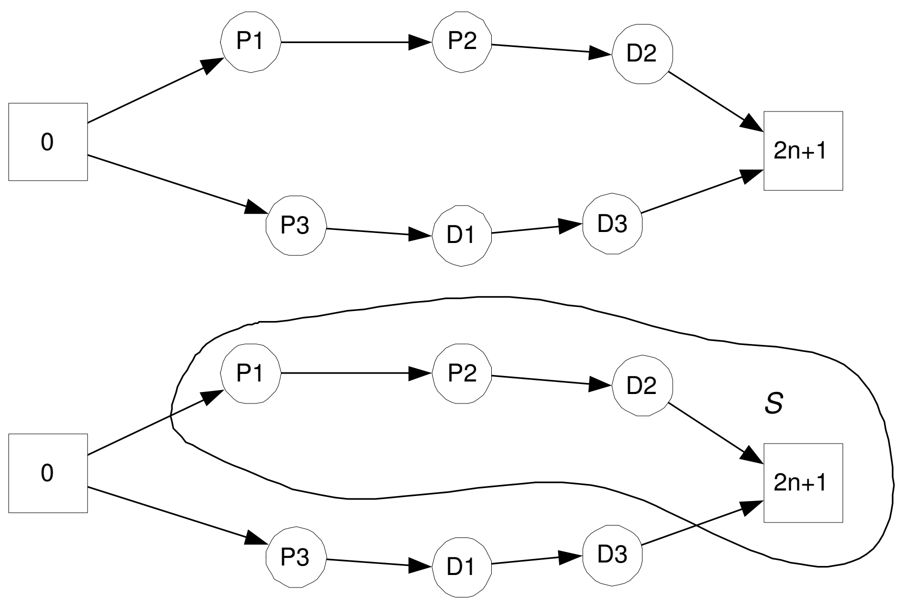
- Problem: The model does not guarantee that the pickup and delivery of a request are performed by the same vehicle. To ensure this we first define the set $\mathcal{S}$ consisting of all node subsets $S \subset N$ such that there is at least one request i for which $i \in S$ but $n + i \notin S$.

- Now the following set of equations (*precedence constraints*) ensure that each pickup/delivery pair is served by the same vehicle.

$$\sum_{i \in S} \sum_{j \in N \setminus S} x_{ij} \geq 1 \quad \forall S \in \mathcal{S}$$

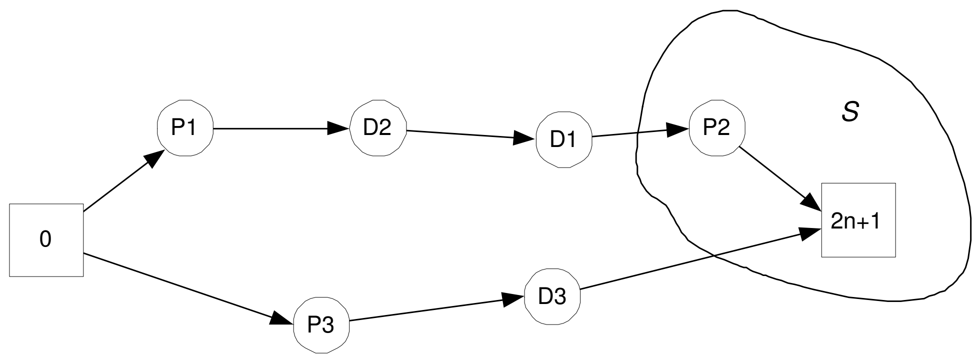
The equation simply express that one edge should leave the set (we have to leave the set in order to visit $n + i$).

Example 1:



Compact model (DARP2)

Example 2 (precedence is also ensured by the constraint):



- New problem: $\mathcal{S}$ grows exponentially with n. Constraints must be generated dynamically.

- Given fractional solution $\bar{x}$ a violated precedence constraint can be found using the following algorithm.

  1. Construct a weighted graph $\bar{G} = (N, \bar{A})$ where $\bar{A} = \{(i, j) \in A; \bar{x_{ij}} > 0$. Each edge (i,j) in $\bar{A}$ has an associated weight $w_{ij} = \bar{x_{ij}}$
  2. **for all i in P do**
     (a) Find the minimum cut between i and $n + i$ in $\bar{G}$
     (b) If the weight of the minimum cut is less than 1 then a violated inequality has been found

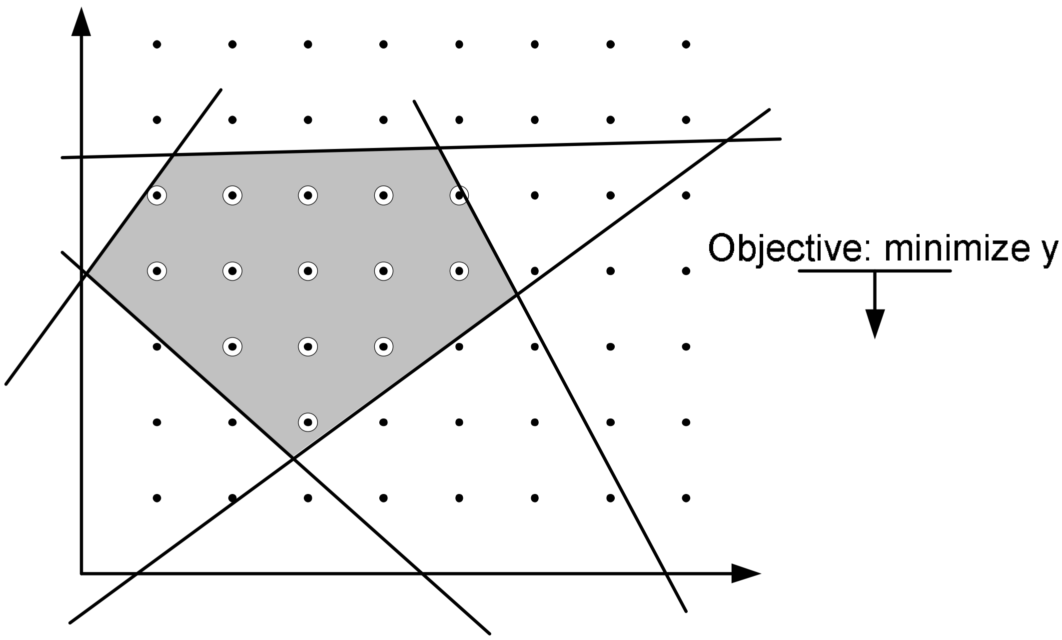
- The correctness of the algorithm follows easily

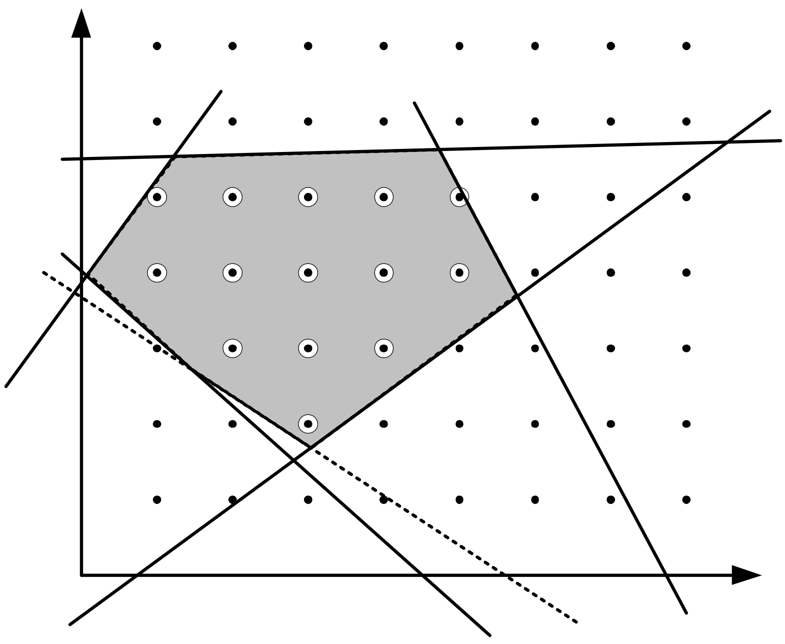
  – If the weight of minimum cut is less than 1 then the cut identifies a set S that violates the inequality
  – If the weight of minimum cut is greater than or equal to 1 for all i then we can show by contradiction that no precedence constraint will be violated.

Comparing DARP1 to DARP2

- DARP1: Certain extra constraints are easier to represent like:
  - Heterogenous fleet
  - Route duration constraints
- DARP1 can be solved directly using CPLEX, DARP2 needs special implementation.
- DARP2 is expected to solve problems faster

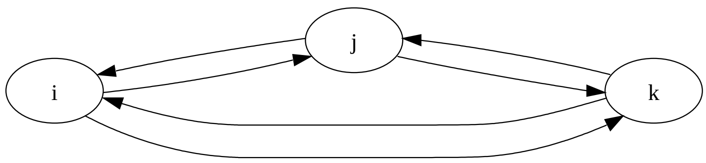
Valid inequalities





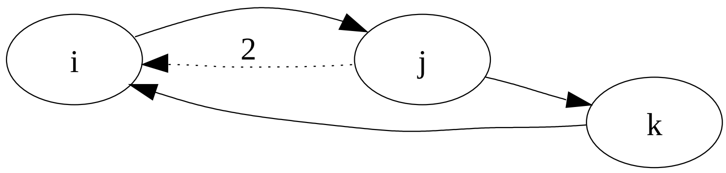
Valid inequalities - some examples

Subtour elimination constraints



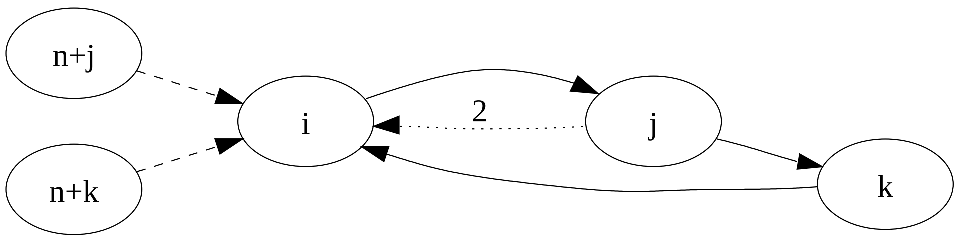
$$x_{ij} + x_{ji} + x_{jk} + x_{kj} + x_{ki} + x_{ik} \leq 2$$

Lifting for directed case:



$$x_{ij} + 2x_{ji} + x_{jk} + x_{ki} \leq 2$$

Lifting for DARP case:

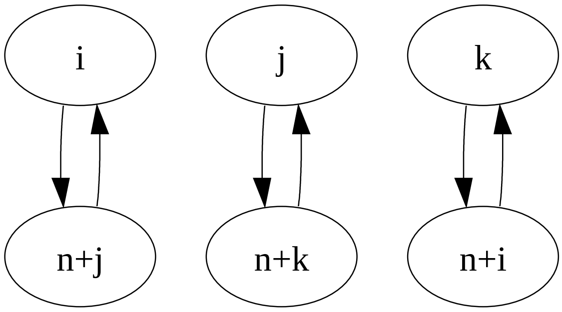


$$x_{ij} + 2x_{ji} + x_{jk} + x_{ki} + x_{n+j,i} + x_{n+k,i} \leq 2$$

General expression and more liftings described in paper.

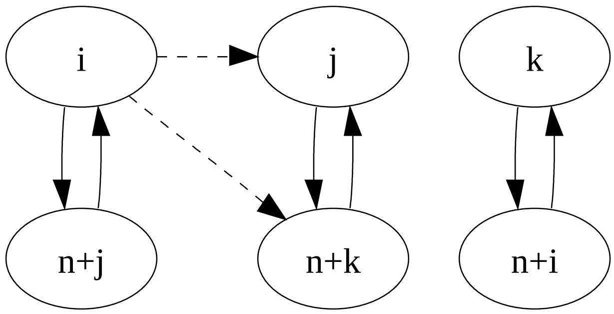
Separation algorithms?

Generalized order constraints



$$x_{i,n+j} + x_{n+j,i} + x_{j,n+k} + x_{n+k,j} + x_{k,n+i} + x_{n+i,k} \leq 2$$

Lifting for directed case:



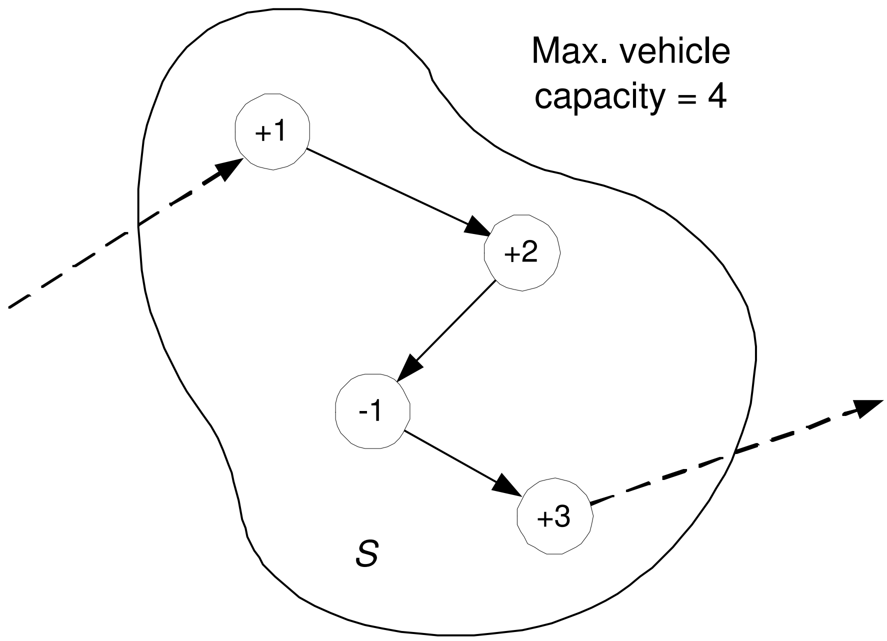
$$x_{i,n+j}+x_{n+j,i}+x_{j,n+k}+x_{n+k,j}+x_{k,n+i}+x_{n+i,k}+x_{ij}+x_{i,n+k} \leq 2$$

Separation algorithms?

Capacity constraints

$$\sum_{i \in S} \sum_{j \in N \setminus S} x_{ij} \geq \left\lceil \frac{q(S)}{Q} \right\rceil \quad \forall S \subseteq P \cup D$$

$q(S) = \sum_{i \in S} q_i$



Separation algorithms?

Infeasible path constraints

If the path $i_1 \to i_2 \to \ldots \to i_h$ is infeasible because of time window or ride time constraints (or a combination) then the following inequality is valid:

$$\sum_{i=1}^{h-1} x_{i,i+1} \leq h - 2$$

Can be separated in polynomial time.

Even more compact model (DARP3)

Using some of the inequalities just presented, we can get rid of the B_i, Q_i and L_i variables.

$$\min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in P$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall j \in P$$

$$\sum_{i \in S} \sum_{j \in N \setminus S} x_{ij} \geq 1 \quad \forall S \in \mathcal{S}$$

Infeasible path inequality that ensures that time window, capacities and ride time constraints are obeyed. $\mathcal{P}$ is the set of all infeasible paths. Each path in $\mathcal{P}$ is stored as a set of edges.

$$\sum_{(i,j) \in E^*} x_{ij} \leq |E^*| - 1 \quad \forall E^* \in \mathcal{P}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in N, j \in N$$

Computational results

See other slide

Conclusion

- A more compact model in terms of number of binary variables was profitable.
- Getting rid of the "superflous" fractional variables didn't improve running time.
- We have just scratched the surface. There are more to tell, and even more to discover.
- Plenty of open algorithmic questions - how to design good separation routines?